Fast and Slow: The Dynamics of Superrotation Phenomena in Planetary Atmospheres
III. Super-rotation in simplified systems

Peter Read

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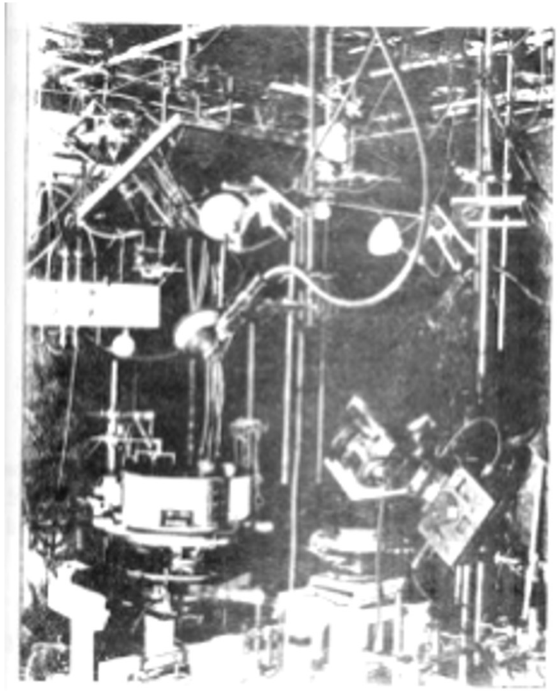
Super-rotation in simplified systems

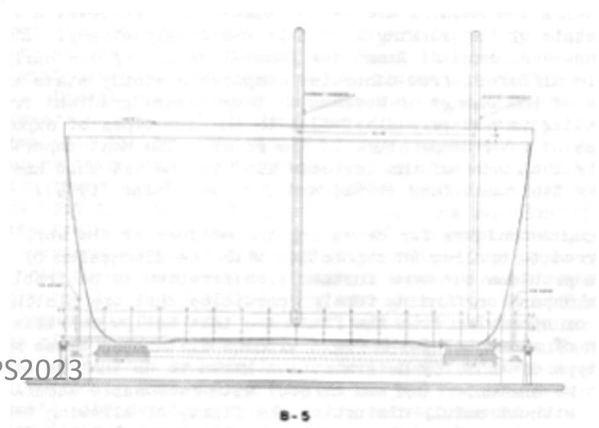
- Laboratory experiments and demonstrations
  - Illustrating mechanisms and basic principles, scaling etc.
- Simplified and intermediate complexity numerical models
  - Circulation regimes & parameter sweeps
  - Zonally symmetric forcing
  - Equatorial jets
  - Thermal tides & tidally locked forcing

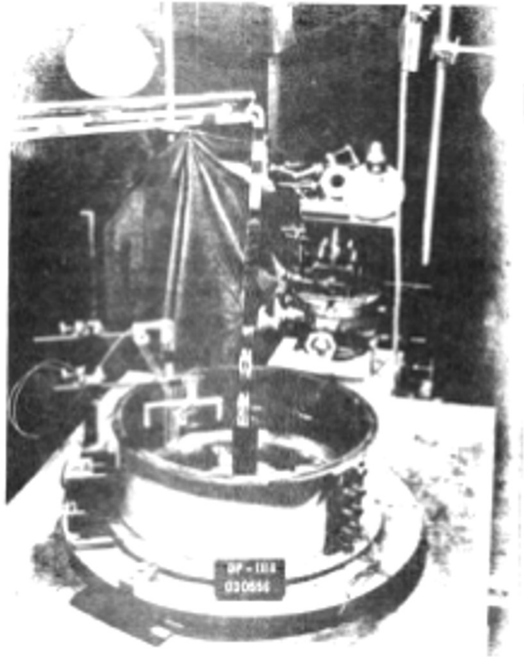
Laboratory experiments: “Dishpan” convection

- Early experiments on rotating convection at University of Chicago [Fultz et al. 1956]
- Open cylinder (“dishpan”), heated from below near the outer edge and cooled at the centre
- Rotated about its axis at Ω rad s^{-1}
- Produces circulations analogous to axisymmetric “Hadley” and baroclinic wavey “Rossby” regimes, depending upon Ω.

Dave Fultz

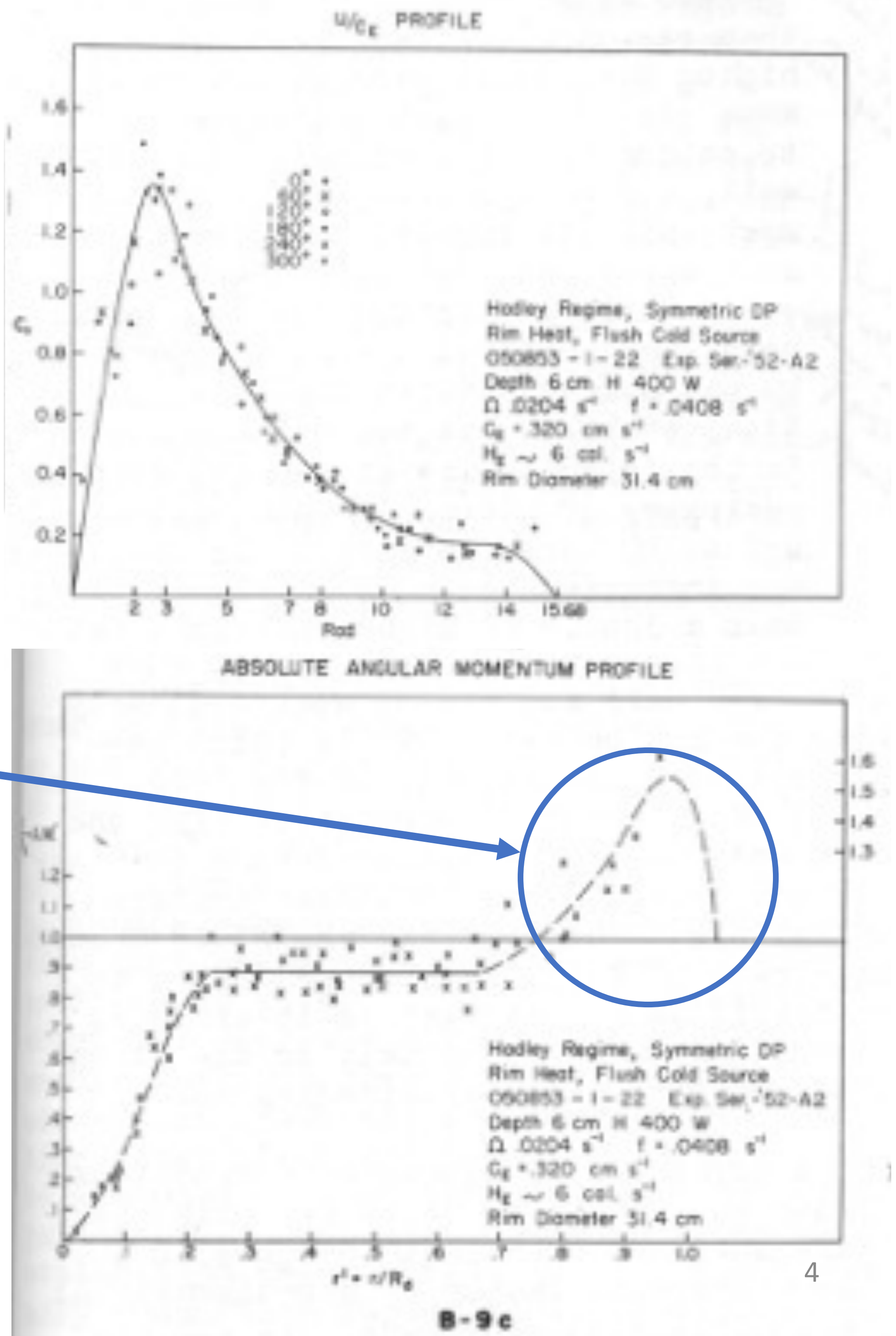






Laboratory experiments: "Dishpan" convection

- Early experiments on rotating convection at University of Chicago [Fultz et al. 1956]
- Axisymmetric flow profile shows evidence of super-rotation?
  - $\mathcal{R} = 2.66$
  - s > 0 near outer rim of tank
  - *m* ~ constant for 0.2 < r/b < 0.7
  - Viscous AM transfer or eddies….?



Laboratory experiments: Rossby wave mixing

- Generation of Rossby waves in a 2 m dia. rotating cylindrical tank [Whitehead 1975]
- Free surface deforms centrifugally to a parabolic interface
  - $h = 2.934 + 0.00126r^2$ (cm)
  - Imposes a "planetary vorticity gradient"
  - $\beta = \dfrac{2\Omega}{h}\dfrac{\partial h}{\partial r}$
- Various localized forcing mechanisms
  - Vertically oscillating plunger
  - Bubbles
  - Radially oscillating plate

Mean flow generated by circulation on a β-plane: An analogy with the moving flame experiment

By JOHN A. WHITEHEAD, JR., *Department of Physical Oceanography, Woods Hole Oceanographic Institution, Woods Hole, Mass. 02543, USA*

(Manuscript received May 31; revised version September 26, 1974)

ABSTRACT

A number of laboratory experiments are described in which water with a curved upper surface in a rotating basin exhibited prograde flows when stirred by stirrers which put no azimuthal torque upon the fluid. It is suggested that the flows were generated by Reynolds stresses of the circulating fluid, and that this is a general consequence of circulations on a β-plane. This is reinforced by an analogy between the equations of the moving flame experiment and the equations of flow on a β-plane. Implications upon atmospheric and oceanic flows are mentioned.

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- Free surface deforms centrifugally to a parabolic interface
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  - Imposes a "planetary vorticity gradient"
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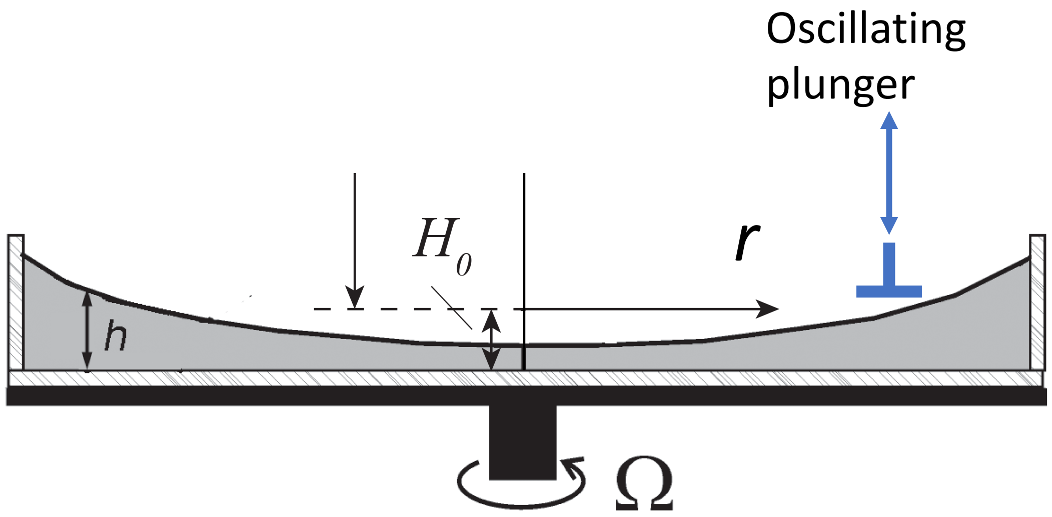


Laboratory experiments: Rossby wave mixing

- Generation of Rossby waves in a 2 m dia. rotating cylindrical tank [Whitehead 1975]
- Various localized forcing mechanisms
  - Vertically oscillating plunger
- Forms a prograde jet at radius of the plunger
  - Weaker retrograde zonal flow either side

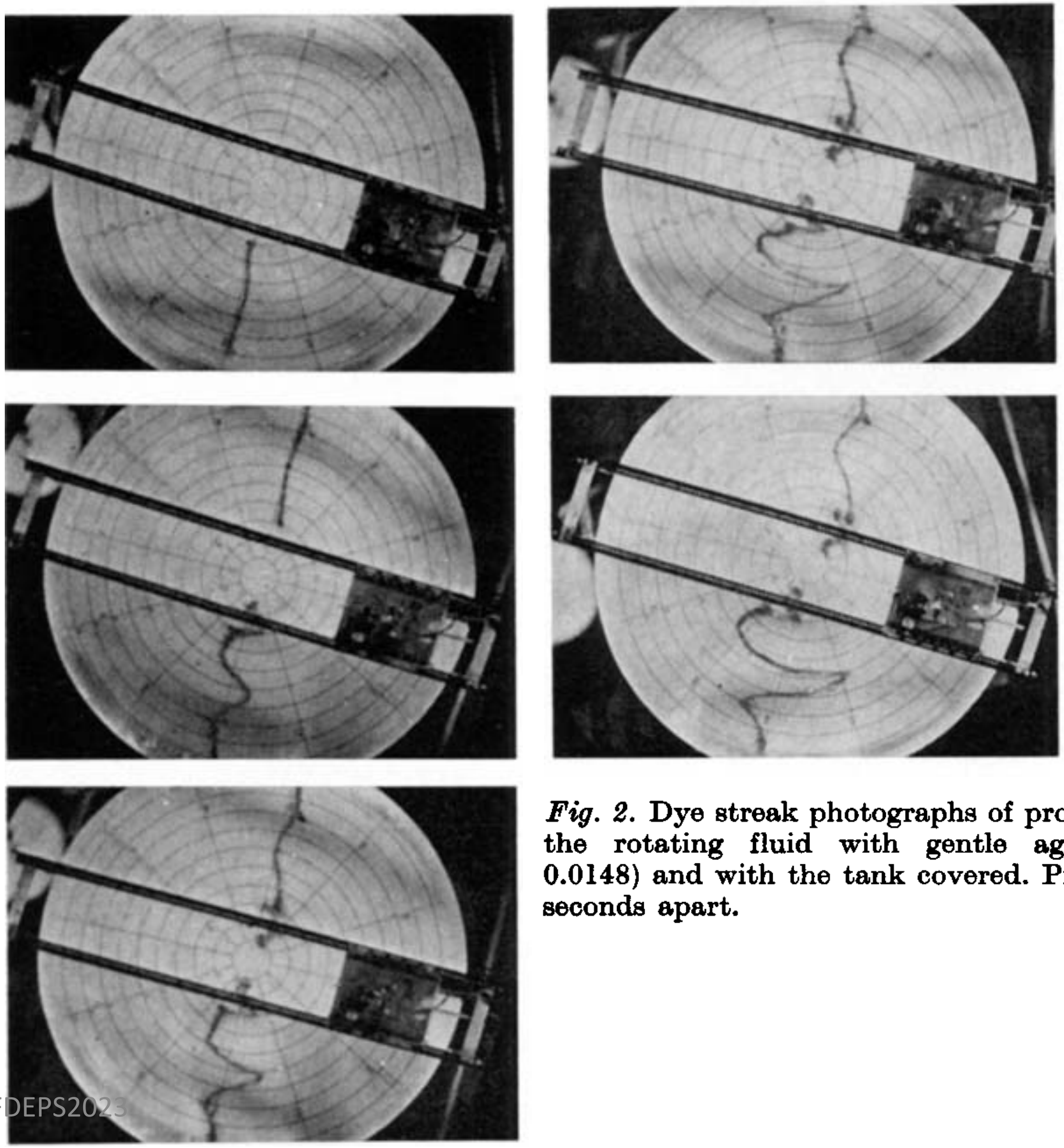


Fig. 2. Dye streak photographs of prograde flow in the rotating fluid with gentle agitation (ε_p = 0.0148) and with the tank covered. Pictures are 32 seconds apart.

Laboratory experiments: Rossby wave mixing

- Generation of Rossby waves in a 2 m dia. rotating cylindrical tank [Whitehead 1975]
- Forms a prograde jet at radius of the plunger
  - Weaker retrograde zonal flow either side
- Strength of jet ($\varepsilon_{jet} = \overline{u_{max}}/\Omega L$) scales roughly with $\varepsilon_{plunger}^2$ at small amplitude
  - Saturates at larger amplitude

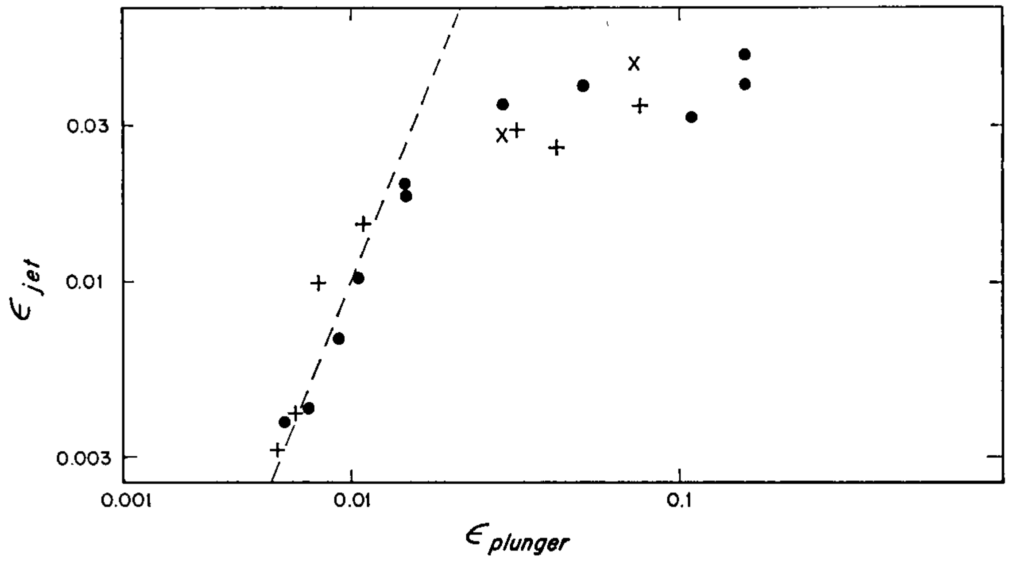
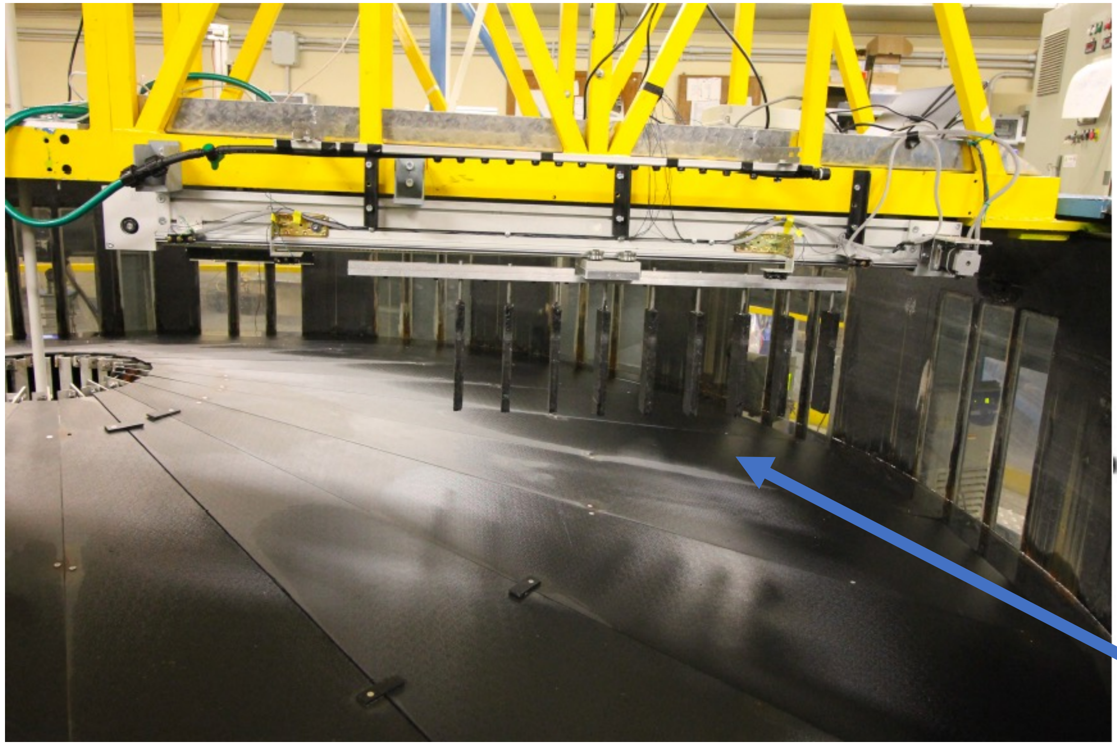
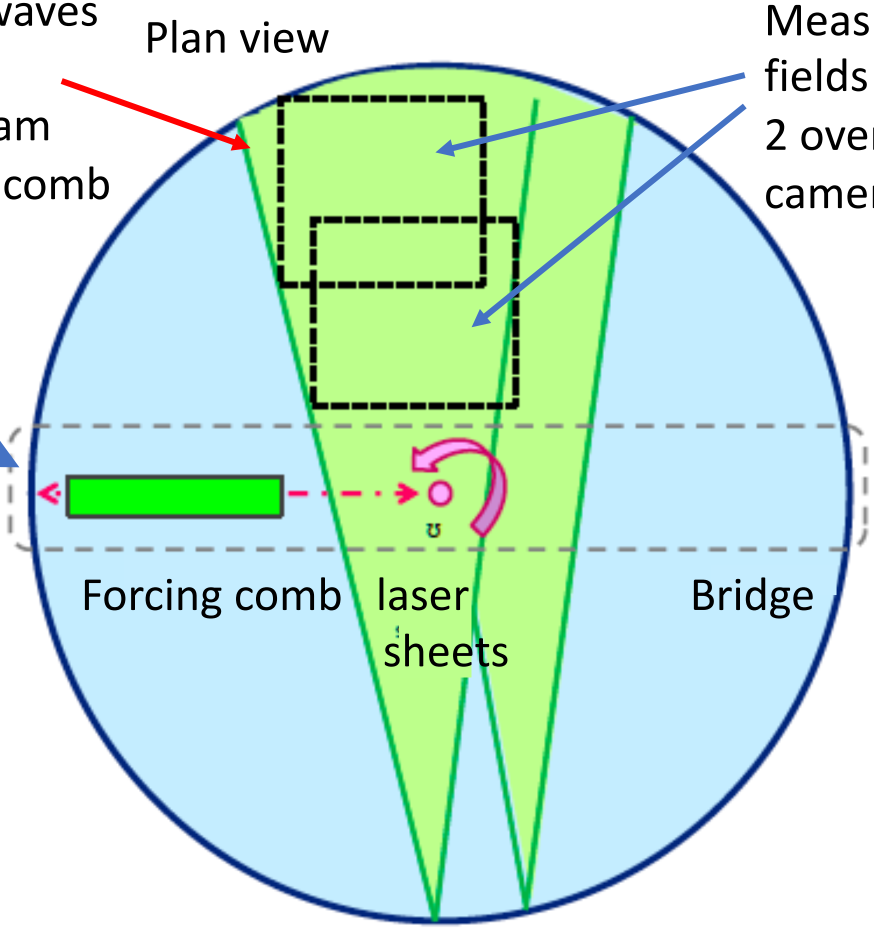


Fig. 3. Rossby number of the flow as a function of Rossby number of the plunger for experiments with a covered tank. Amplitude (peak to peak): X − 1.75 cm, −3.5 cm, +5.20 cm. A dashed line with a slope of 2:1 is inserted for comparison.

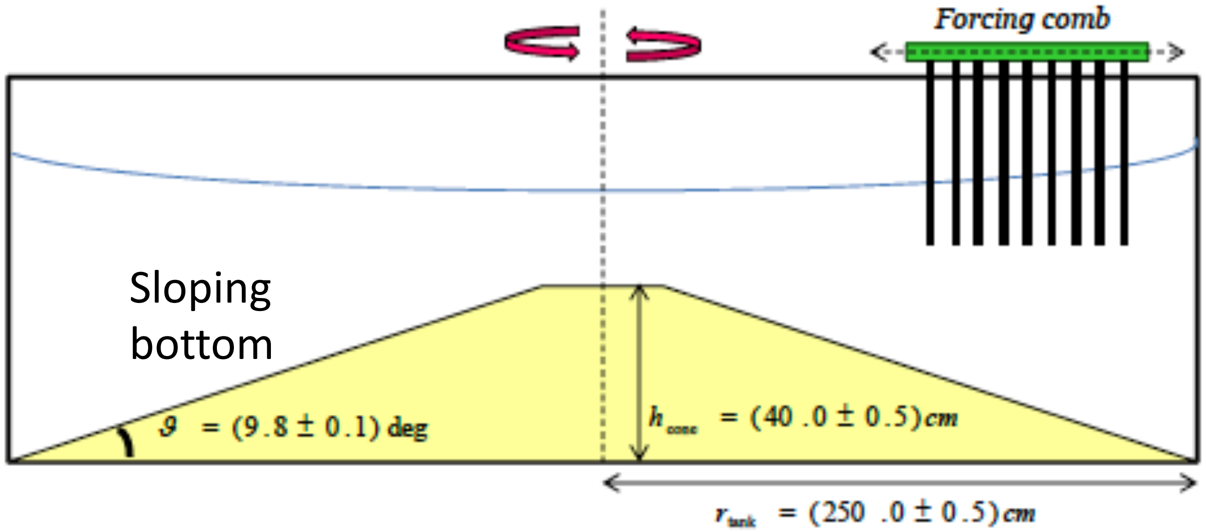
Experimental concept (Turlab – Torino, Italy)



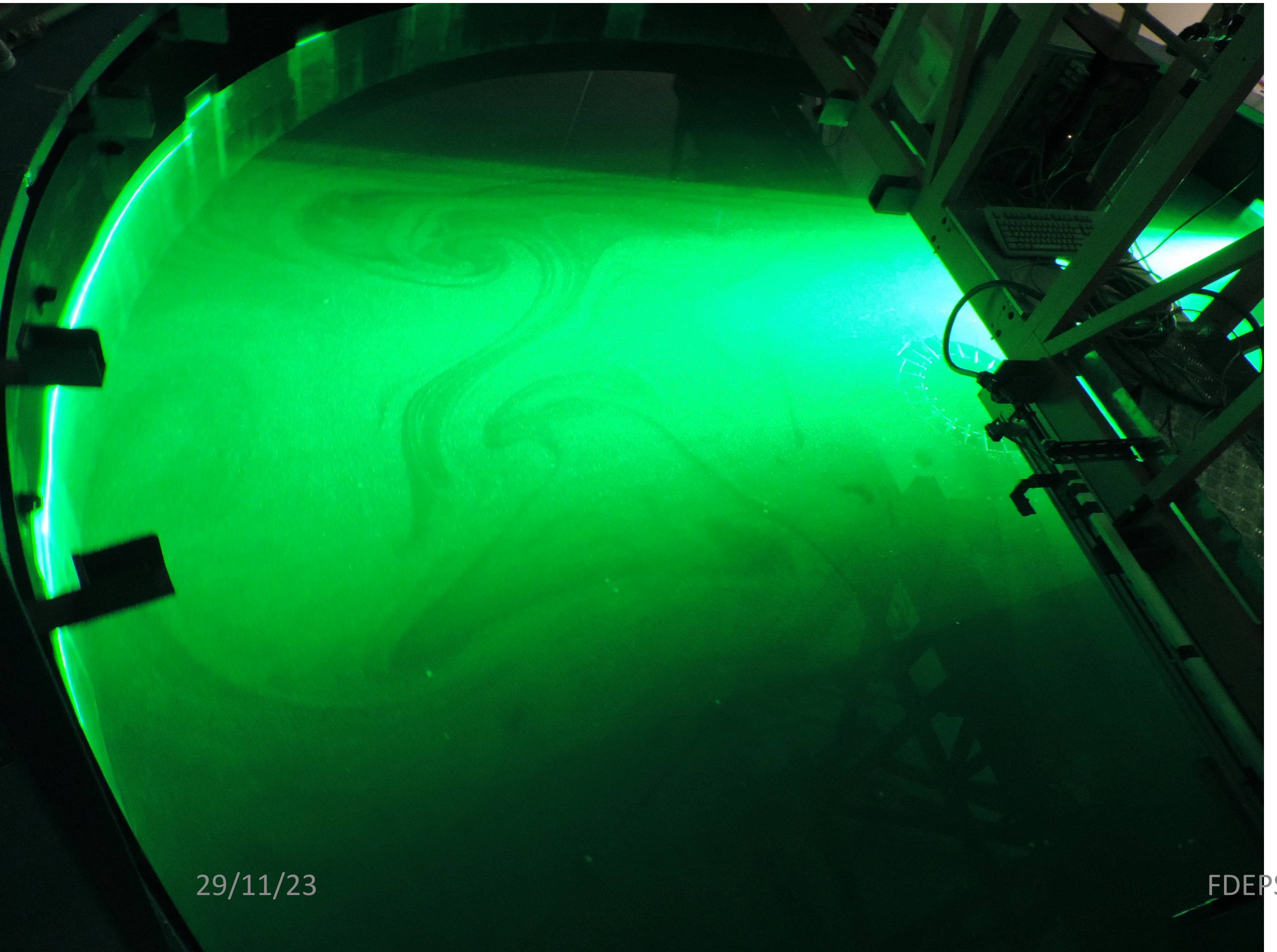
Observe waves and jets downstream of forcing comb

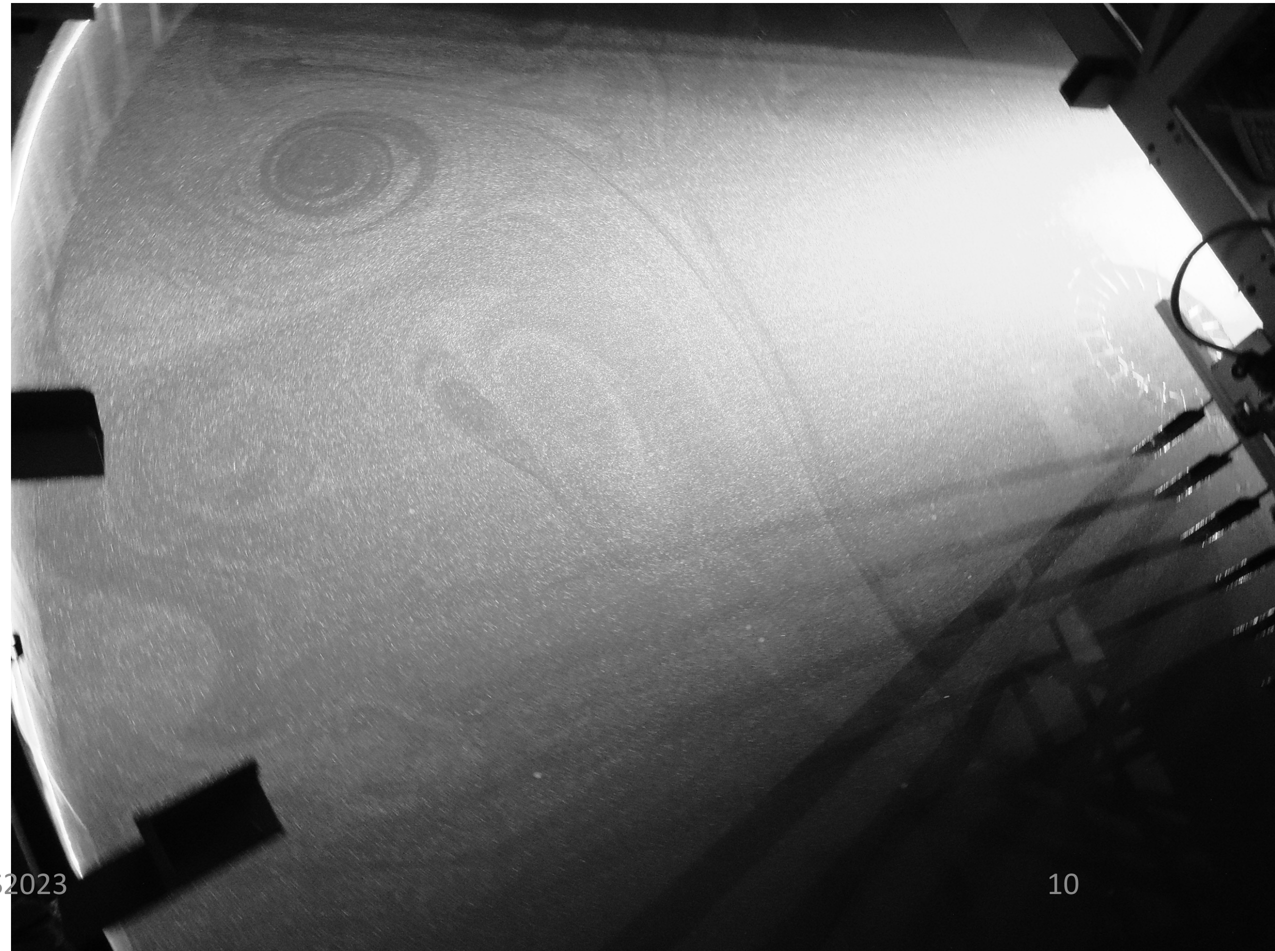


Measure velocity fields via PIV in 2 overlapping camera views



Chaotic tracer patterns indicate zonal jets, Rossby waves and compact vortices





$R_\beta \approx 2.25$

Waves, Turbulence and Diffusion in Beta-Plumes: Rotating tank experiments at TurLab, Turin

Team Beta-WTD

EXPT09, 10 February 2017, CIV2, resolution ~ 1.4 cm, rotation 30.3 s, comb 7.5 cm s^{-1}

Potential vorticity and velocity

80x real time

$R_\beta \approx 2.20$

Waves, Turbulence and Diffusion in Beta-Plumes: Rotating tank experiments at TurLab, Turin

Team Beta-WTD

EXPT07, 8 February 2017, CIV2, resolution ~ 1.4 cm, rotation 61 s, comb 7.5 cm s^{-1}

Potential vorticity and velocity

80x real time

$R_\beta \approx 2.18$

Waves, Turbulence and Diffusion in Beta-Plumes: Rotating tank experiments at TurLab, Turin

Team Beta-WTD

EXPT06, 7 February 2017, CIV2, resolution ~ 1.4cm, rotation 119 s, comb 7.5 cm s^{-1}

Potential vorticity and velocity

80x real time

Potential vorticity $q = \dfrac{\boldsymbol{k}.\boldsymbol{\nabla}\times\boldsymbol{u}+2\boldsymbol{\Omega}}{h}$

Zonal mean flow: time variations

τ_R (s)

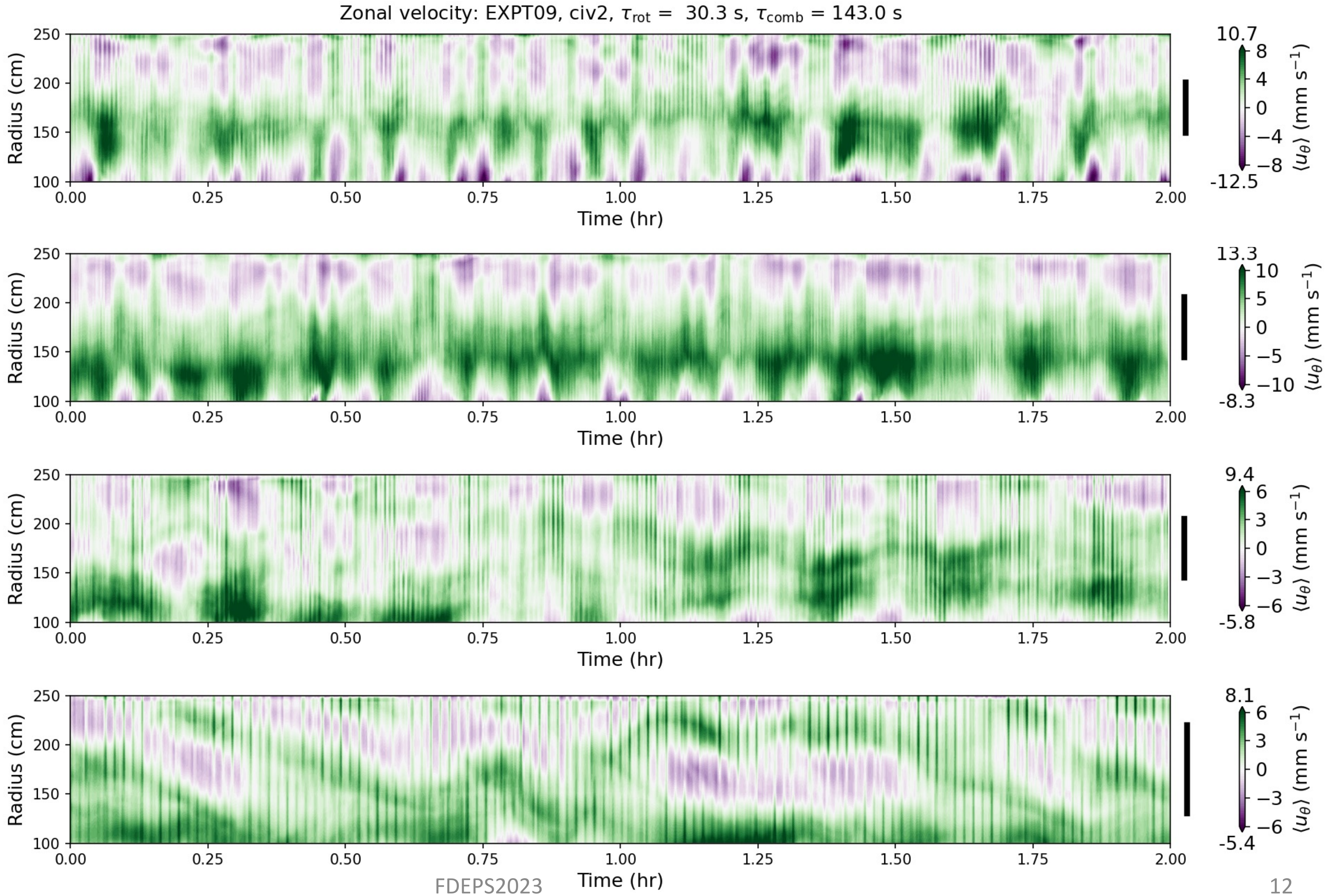
30.3

- Width of mean zonal flow increases with τ_R – scales with Rhines scale?
- Timescale of Fluctuations also Increases with τ_R

40.4

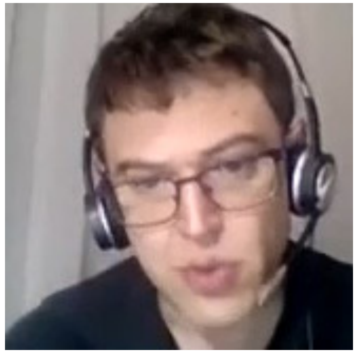
61.0

119.0



Laboratory experiments: Internal wave streaming

- Generation of a mean flow by dissipation of vertically propagating internal gravity waves [Semin et al. (2016,2018)]
- A single, progressive or standing gravity wave excited at the upper boundary of a cylindrical tank of salt-stratified fluid
- Generate a mean azimuthal flow as the wave propagates downwards and dissipates



Benoit Semin Francois Petrelis. Stephan Fauve

PHYSICS OF FLUIDS **28**, 096601 (2016)

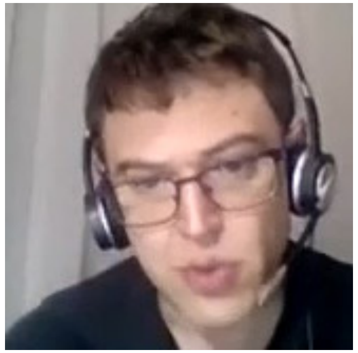
Generation of a mean flow by an internal wave

B. Semin, G. Facchini, F. Pétrélis, and S. Fauve
Laboratoire de Physique Statistique, École Normale Supérieure, PSL Research University, Université Paris Diderot Sorbonne Paris-Cité, Sorbonne Universités UPMC Univ Paris 06, CNRS, 24 Rue Lhomond, 75005 Paris, France

(Received 14 March 2016; accepted 1 September 2016; published online 22 September 2016)

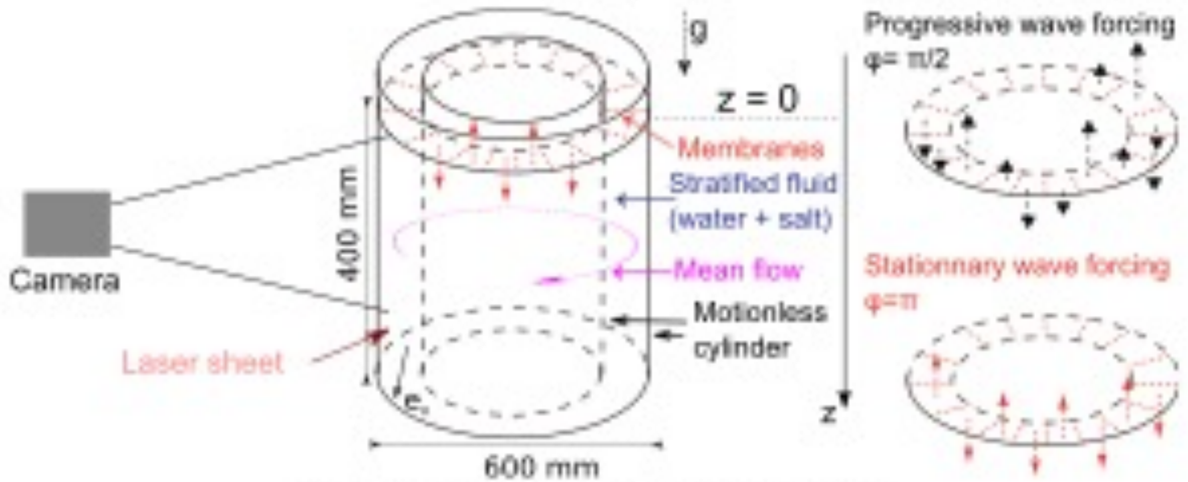
We experimentally study the generation of a mean flow by a two-dimensional progressive internal gravity wave. Due to the viscous damping of the wave, a non-vanishing Reynolds stress gradient forces a mean flow. When the forcing amplitude is low, the wave amplitude is proportional to the forcing and the mean flow is quadratic in the forcing. When the forcing amplitude is large, the mean flow decreases the wave amplitude. This feedback saturates both the wave and the mean flow. The profiles of the mean flow and the wave are compared with a one-dimensional analytical model. Decreasing the forcing frequency leads to a wave and a mean flow localized on a smaller height, in agreement with the model. *Published by AIP Publishing.* [http://dx.doi.org/10.1063/1.4962937]

Laboratory experiments: Internal wave streaming



Benoit Semin   Francois Petrelis. Stephan Fauve

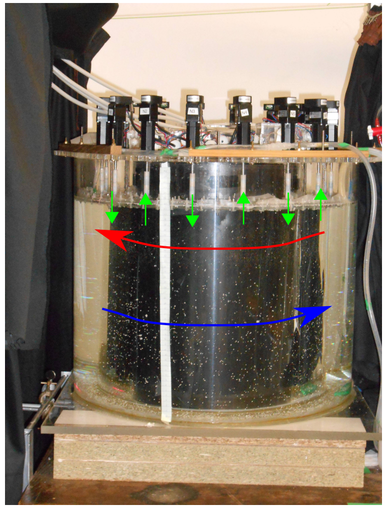
Experimental setup (1)

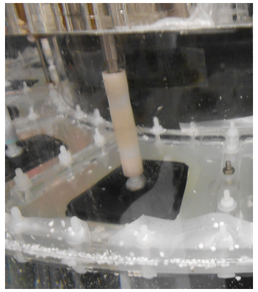


Plumb and McEwan (1978) , Otobe et al. (1998)

- Väisälä-Brunt angular frequency $N = \sqrt{-\frac{\mathrm{d}\rho_0}{\mathrm{d}z}\frac{g}{\rho_0}} \sim 1.5 - 2.2\ \mathrm{rad \cdot s^{-1}}$
- Forcing period $T \sim 15 - 38$ s.
- Forcing amplitude: $M \leq 15$ mm.
- Visualisation : particles, laser sheet. Particle image velocimetry (PIV)

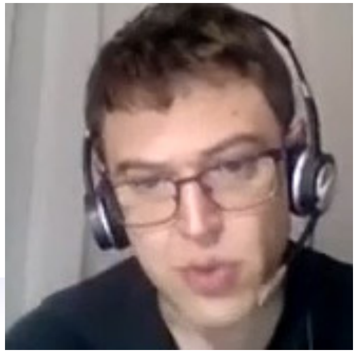
Experimental setup (2)



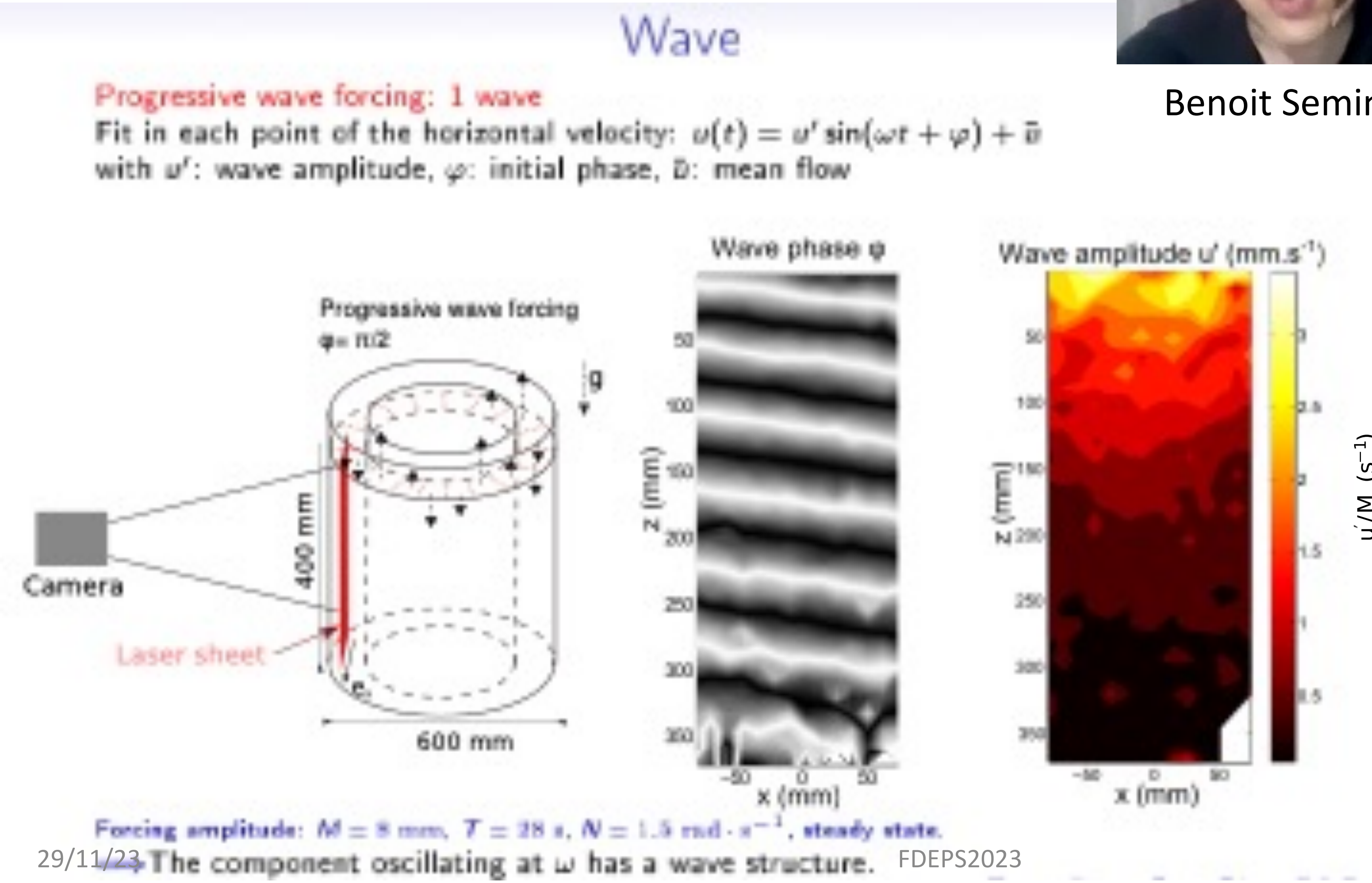


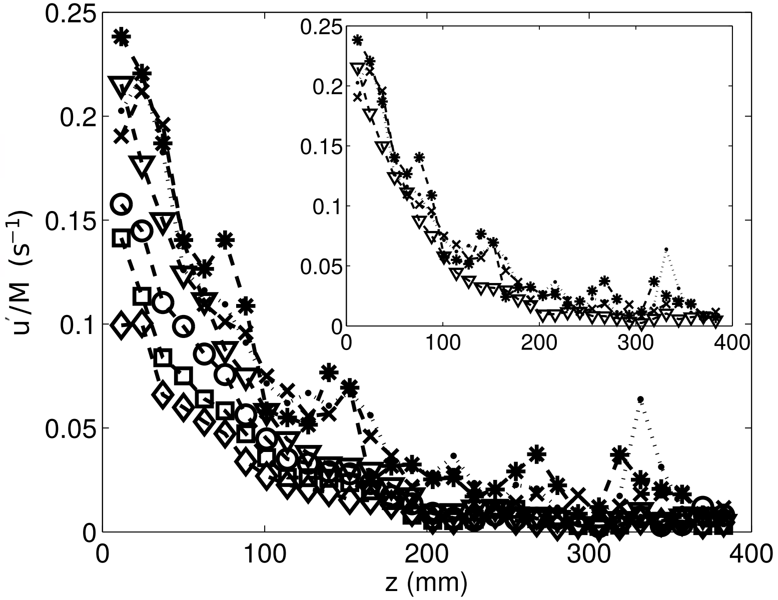
- Each membrane is controlled individually
- Continuous injection of fluid at the bottom and sucking of the fluid at the top (very small velocity $< 10\ \mu\mathrm{m \cdot s^{-1}}$): linear stratification despite of the mixing by the membrane motion.

Laboratory experiments: Internal wave streaming



Benoit Semin   Francois Petrelis. Stephan Fauve

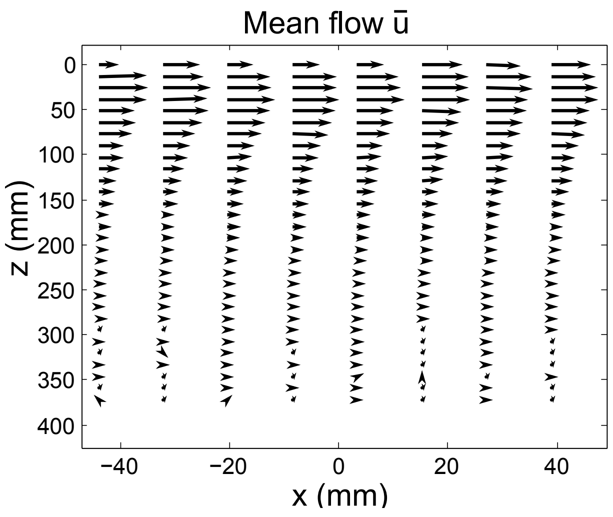




Laboratory experiments: Internal wave streaming

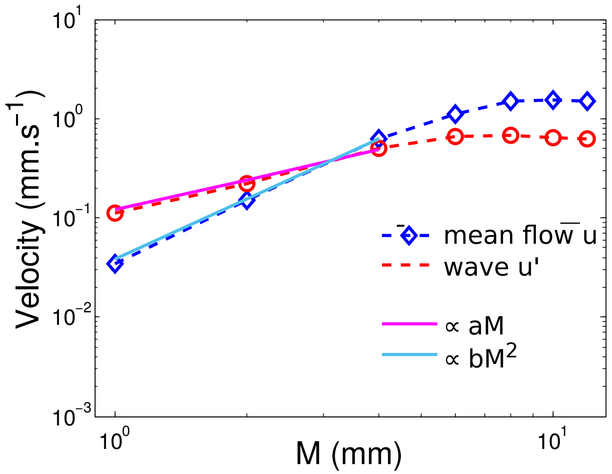
Mean flow

Fit in each point of the horizontal velocity: $u(t) = u' \sin(\omega t + \varphi) + \bar{u}$
with u': wave amplitude, φ: initial phase, $\bar{u}$: mean flow

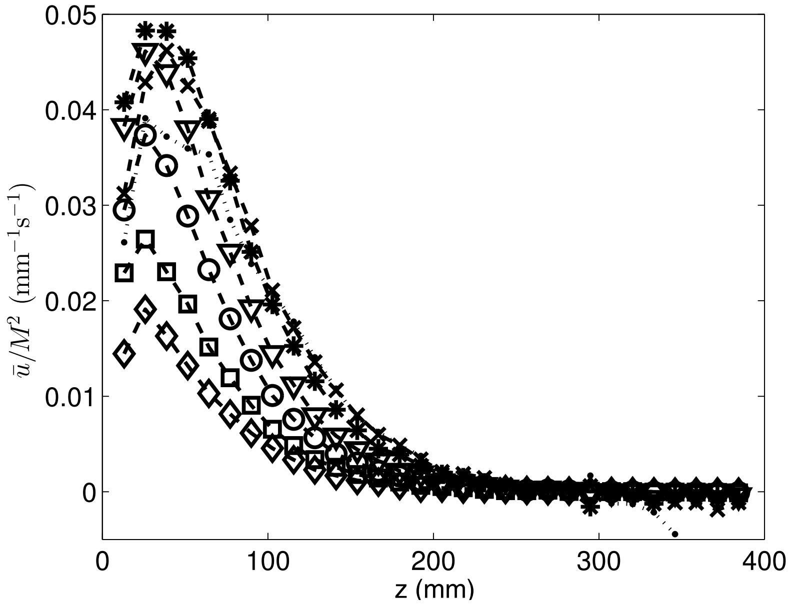


Forcing amplitude: $M = 8$ mm, $T = 28$ s, $N = 1.5\ \mathrm{rad \cdot s^{-1}}$ **steady state.**

Analysis using Fourier Transform.



Forcing amplitude: $T = 38$ s, $N = 1.5\ \mathrm{rad \cdot s^{-1}}$, $z = 64$ mm, **steady state.**



$\Longrightarrow$Always a mean flow.
$\Longrightarrow$The mean flow is homogeneous in the x direction (2D).
$\Longrightarrow$Mean flow proportional to the square of the wave at low M.

Laboratory experiments: Internal wave streaming

Variation with forcing amplitude M

- Navier-Stokes equation (3D):

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\mathbf{v}\right)=-\nabla p+\eta\Delta\mathbf{v}+\rho\mathbf{g}$$

- Velocity: $\mathbf{u}=(u'+\bar{u})\mathbf{e_x}+(v')\mathbf{e_z}$
- Model from Navier-Stokes:

$$\frac{\partial \bar{u}}{\partial t}=-\frac{\partial F}{\partial z}+\nu\frac{\partial^2\bar{u}}{\partial z^2}-\gamma\bar{u}$$

with ν: kinematic viscosity
γ: wall friction

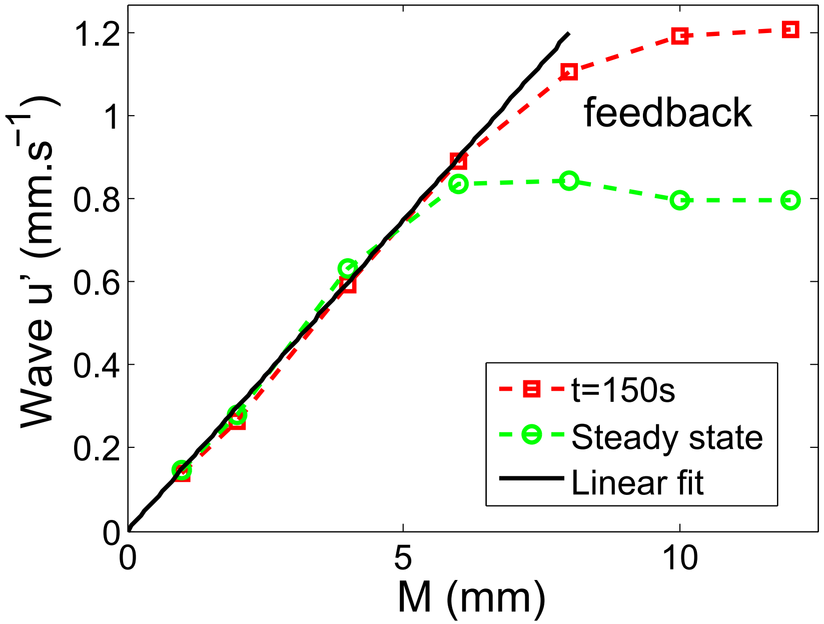
$$F=\overline{u'v'}$$

with v' the vertical wave component

- Prediction of the model: in steady state, without feedback of the mean flow on the wave: $\bar{u}\propto(u')^2$.
In agreement with experimental data.

Laboratory experiments: Internal wave streaming

Feedback



Forcing amplitude: $T = 38$ s, $N = 1.5$ rad $\cdot$ s^{-1}, $z = 64$ mm.

- Model from Navier-Stokes, mass conservation, in Boussinesq and WKB approximations (from Plumb and McEwan 1978):

$$F(\xi) = F(0) \times \exp\left(-\int_0^{\xi}\left[\frac{1}{(1-\bar{U})^4}\right]\mathrm{d}\xi\right)$$

$$
\begin{aligned}
F &= \overline{u'\,v'} \\
c &= \frac{\omega}{k_x} \quad \text{horizontal phase velocity} \\
d &= \frac{k_x c^4}{N^3 \nu} \quad \text{vertical dissipation length} \\
\xi &= \frac{z}{d} \\
\bar{U} &= \frac{\bar{u}}{c}
\end{aligned}
$$

$\Longrightarrow$Negative feedback of mean flow on wave

$\Longrightarrow$Feedback increases the momentum transfer from wave to mean flow at $z = 0$

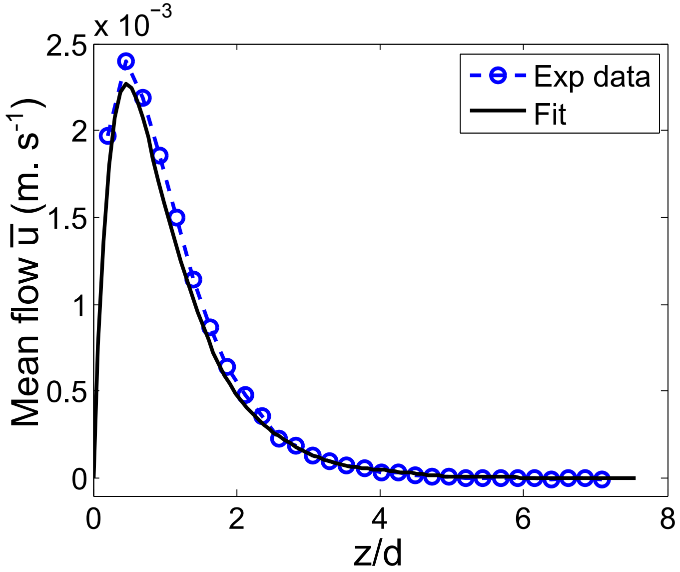
Laboratory experiments: Internal wave streaming

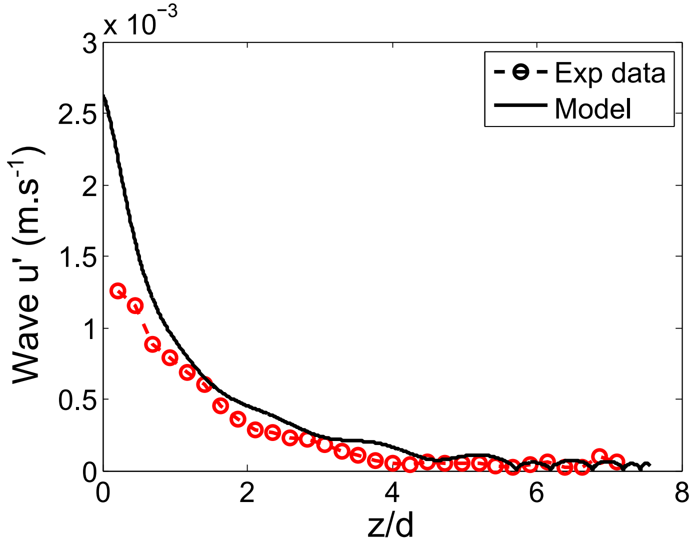
Comparison with the model

$$\frac{\partial \bar{U}}{\partial t} = -d\,\frac{\partial F}{\partial \xi} + \nu d^2 \frac{\partial^2 \bar{U}}{\partial \xi^2} - \gamma \bar{U}$$

$$F(\xi) = F(0) \times \exp\left(-\int_0^{\xi} \left[\frac{1}{(1-\bar{U})^4}\right] \mathrm{d}\xi\right)$$

Comparison with the model. One single fitting parameter $F(0)$.



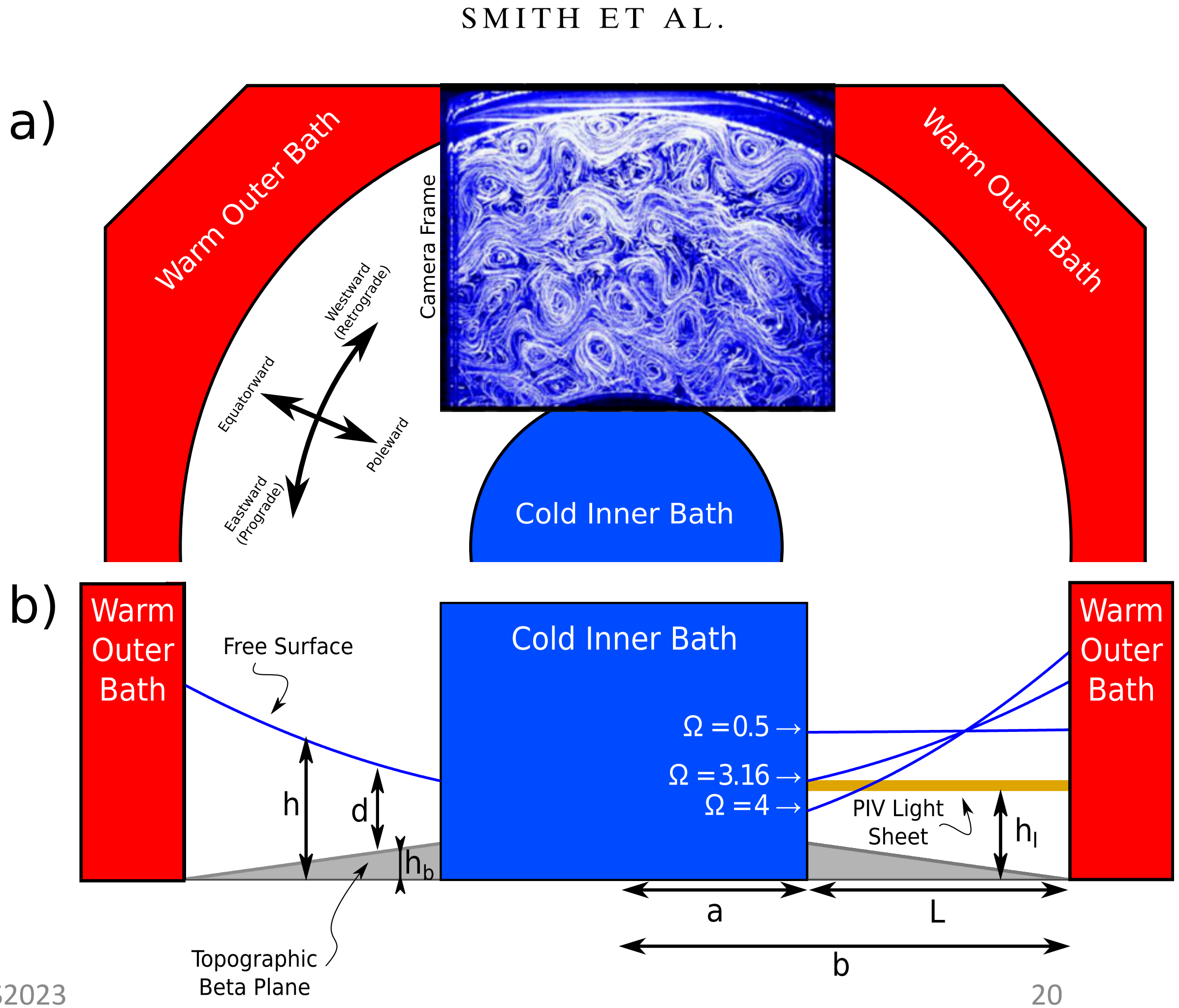


Forcing amplitude: $T = 38$ s, $M = 8$ mm, $N = 1.5$ rad $\cdot$ s^{-1}, **steady state.**

Good agreement between the model and experimental data

Laboratory experiments: Jets driven by [baroclinic] instabilities

- Baroclinically unstable flow in a differentially heated, rotating annulus
  - Sloping bottom and free surface create topographic β-effect
  - Large diameter (1m) annulus [Smith et al. 2014]
- Sources of Rossby-like wave in a rotating, stratified fluid
  - So ... $\overline{u'v'} \neq 0$
  - Dri ... d ret



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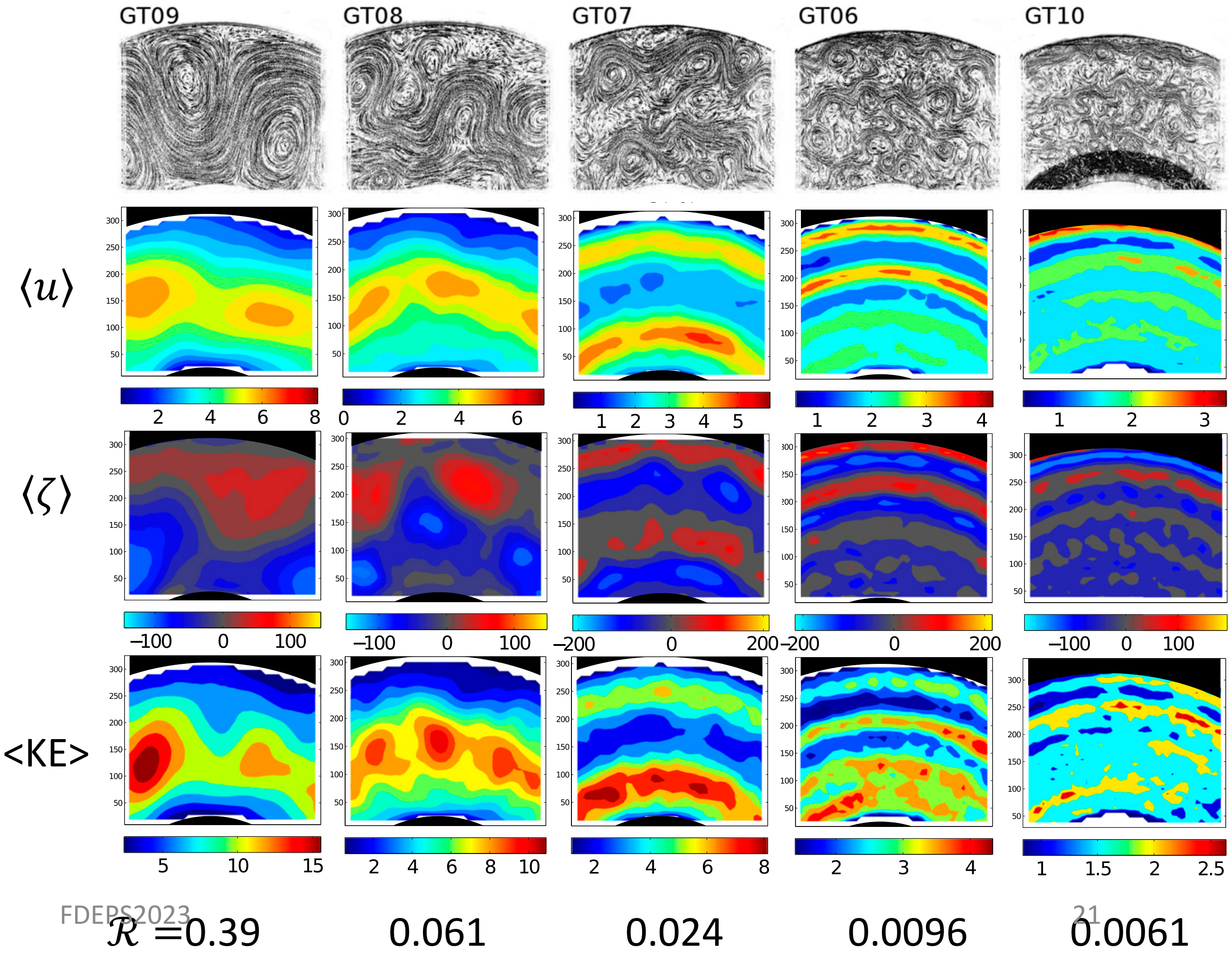
- Sources of Rossby-like wave in a rotating, stratified fluid
  - Horizontal momentum flux $\overline{u'v'} \neq 0$
  - $\boldsymbol{\nabla}.\boldsymbol{F} \approx \frac{1}{r}\frac{\partial}{\partial r}\left(\overline{u'v'}\right)$ drives alternating prograde and retrograde mean flows



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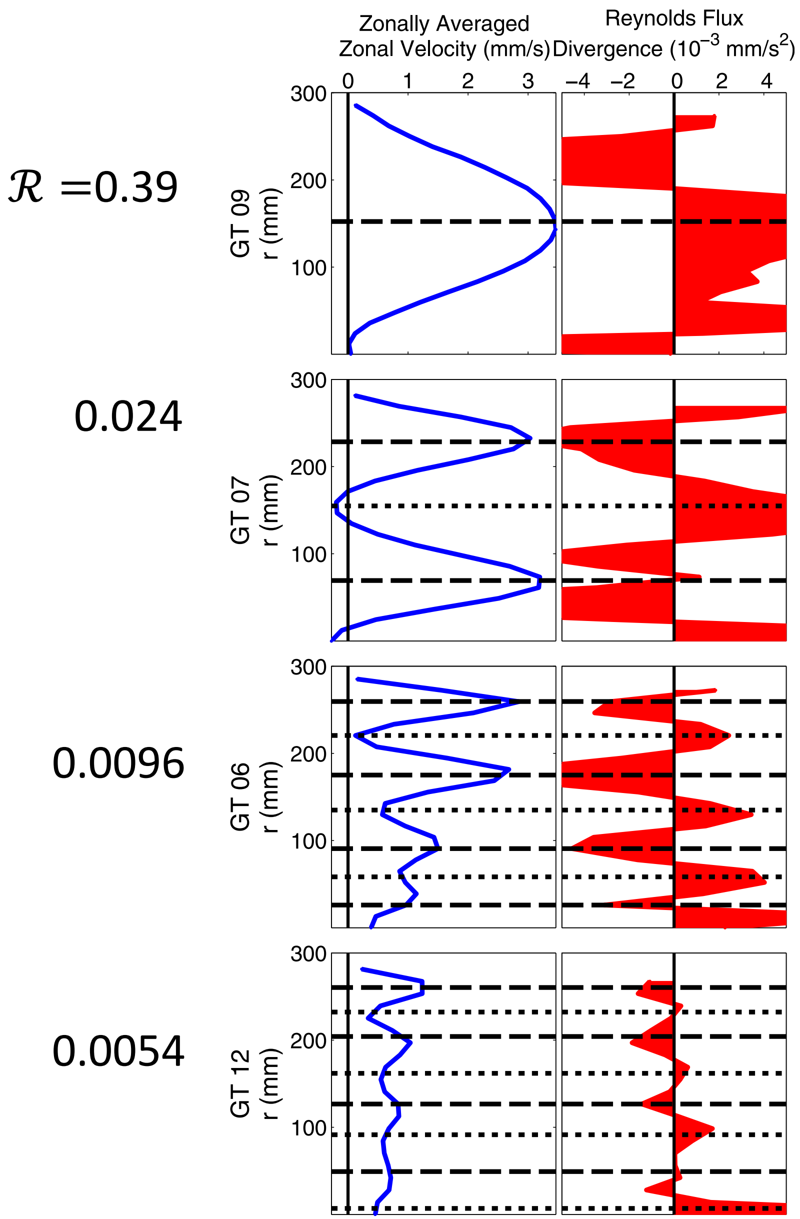
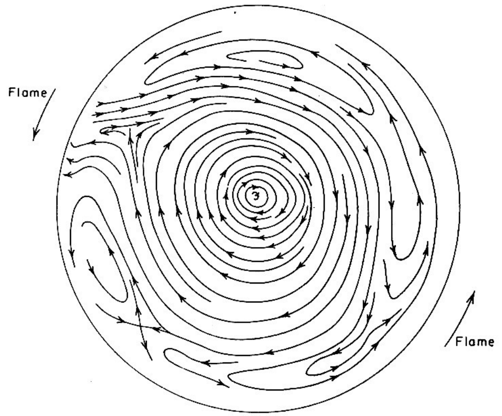


Laboratory experiments: Moving flames and thermal tides?

- Possible sources of vertically propagating waves or tilted convection
  - So vertical momentum flux $\overline{u'w'} \neq 0$
  - Drives a retrograde mean flow close to forcing
- Inspired by the moving flame experiment [Fultz et al. 1956]?

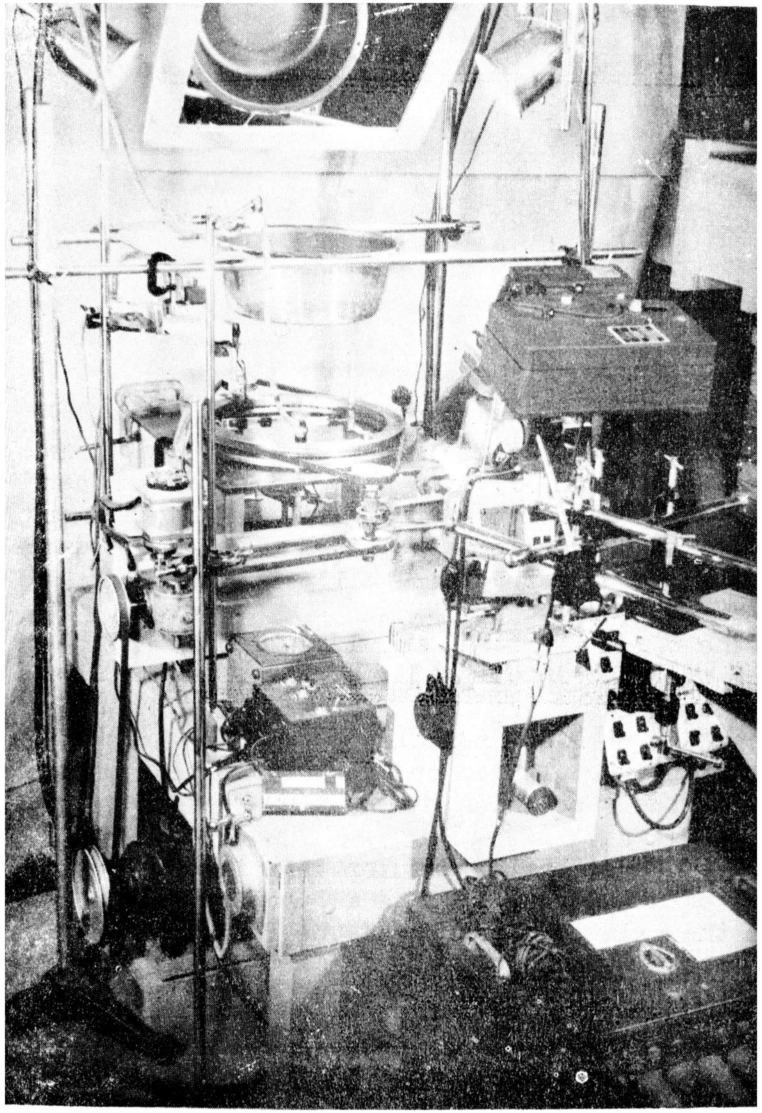
B-20a



DISHPAN MOVING FLAME
201251 - 3 - 13 Streamlines

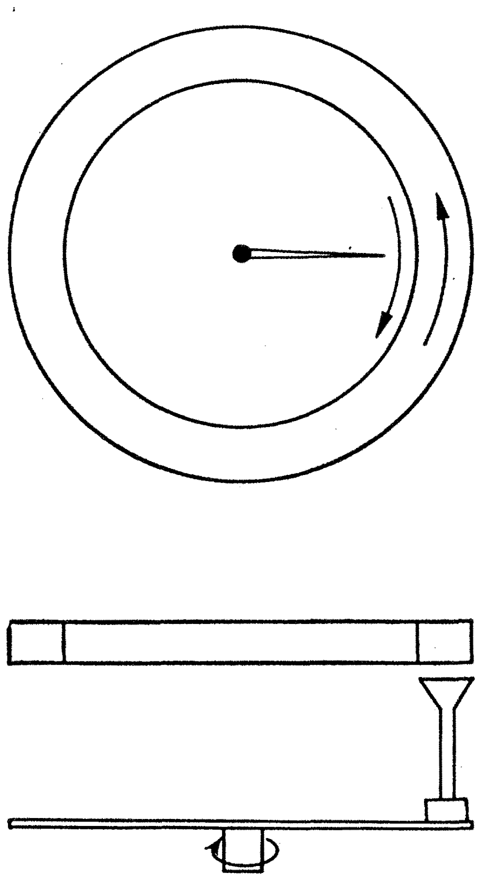
6 cm depth, Rim diameter 31.4 cm
Flame rotation .521 rpm
c_{flame} (at rim radius) 86 cm / sec
Exposure time 4 sec
Fluid velocity (at r′ = .3) .51 cm/sec from line A

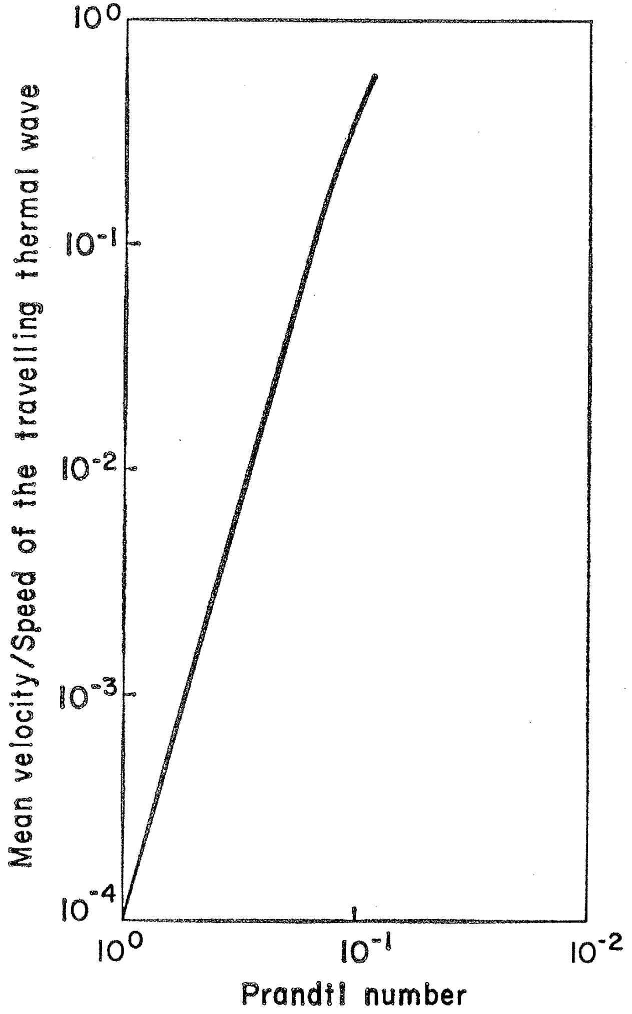
B-20b



Laboratory experiments : Moving flames and thermal tides?

- Possible sources of vertically propagating waves or tilted convection
  - So vertical momentum flux $\overline{u'w'} \neq 0$
  - Drives a retrograde mean flow close to forcing
- Inspired by the moving flame experiment [Fultz et al. 1956]?
- Extended by Schubert & Whitehead [1969] using mercury as working fluid





Laboratory experiments : Moving flames and thermal tides?

- Possible sources of vertically propagating waves or tilted convection
  - So vertical momentum flux $\overline{u'w'} \neq 0$
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- Inspired by the moving flame experiment [Fultz et al. 1956]?
- Extended by Douglas et al. [1972] to use a moving internal heat source

Motion due to a moving internal heat source 471

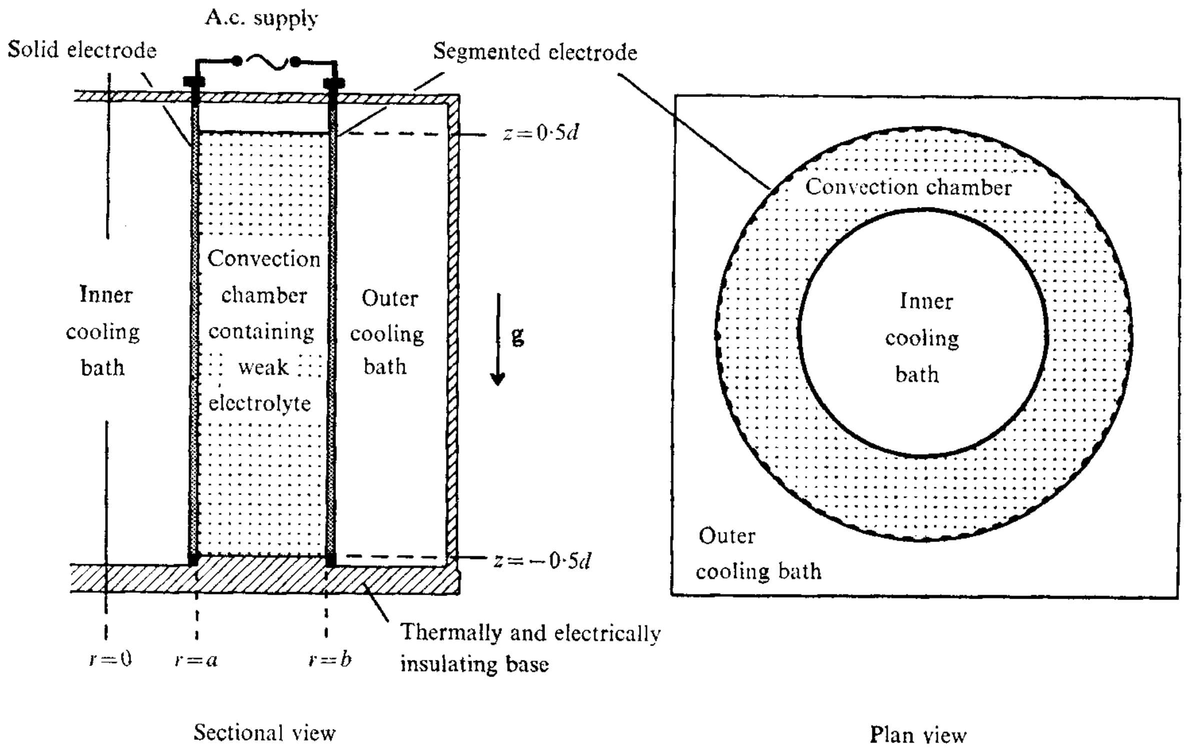
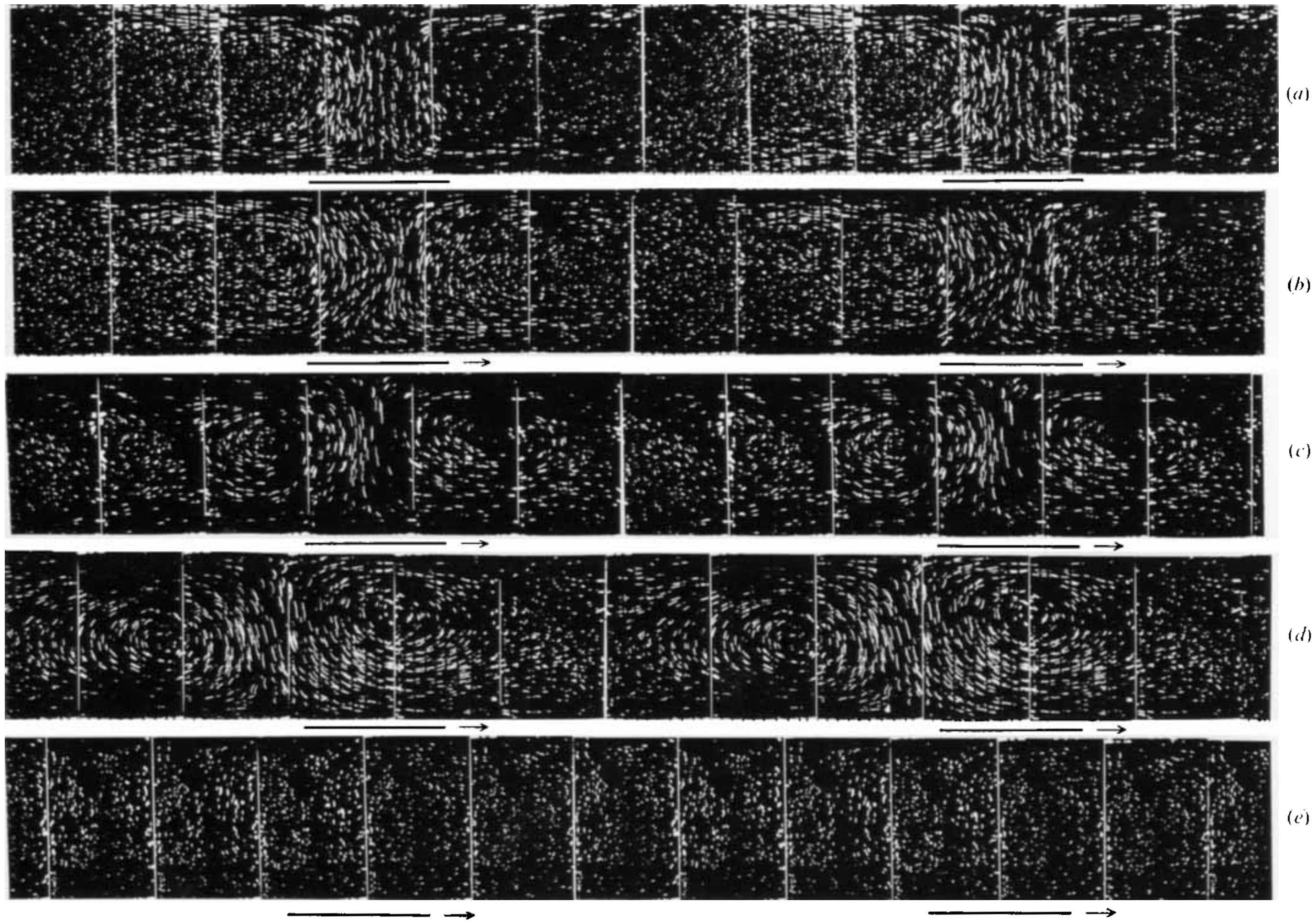


FIGURE 1. Schematic diagram of the apparatus showing the position of the two electrodes. The fluid was heated by passing an alternating current between the electrodes, and cooled by passing water through the two cooling baths. The working fluid occupies the region $a < r < b$ and $-0{\cdot}5d < z < 0{\cdot}5d$.

Laboratory experiments : Moving flames and thermal tides?

- Possible sources of vertically propagating waves or tilted convection
  - So vertical momentum flux $\overline{u'w'} \neq 0$
  - Drives a retrograde mean flow close to forcing
- Inspired by the moving flame experiment [Fultz et al. 1956]?
- Extended by Douglas et al. [1972] to use a moving internal heat source
  - Clearly demonstrates tilted convection cells



Simplified theoretical and numerical models: thermal tides


Stephen B. Fels


Richard Lindzen

- Idealise a thermal tide as the excitation of internal gravity waves by a moving heat source *J'*
  - Fels & Lindzen [1974]
- Inspired by moving flame experiments of Fultz et al. [1956] and later work AND observations of Venus's super-rotation
- Aims to explain and quantify:
  - Acceleration of mean zonal flow in opposite direction to movement of heat source/diurnal heating
  - Magnitude of acceleration and its dependence on atmospheric parameters

Simplified theoretical and numerical models: thermal tides

Richard Lindzen

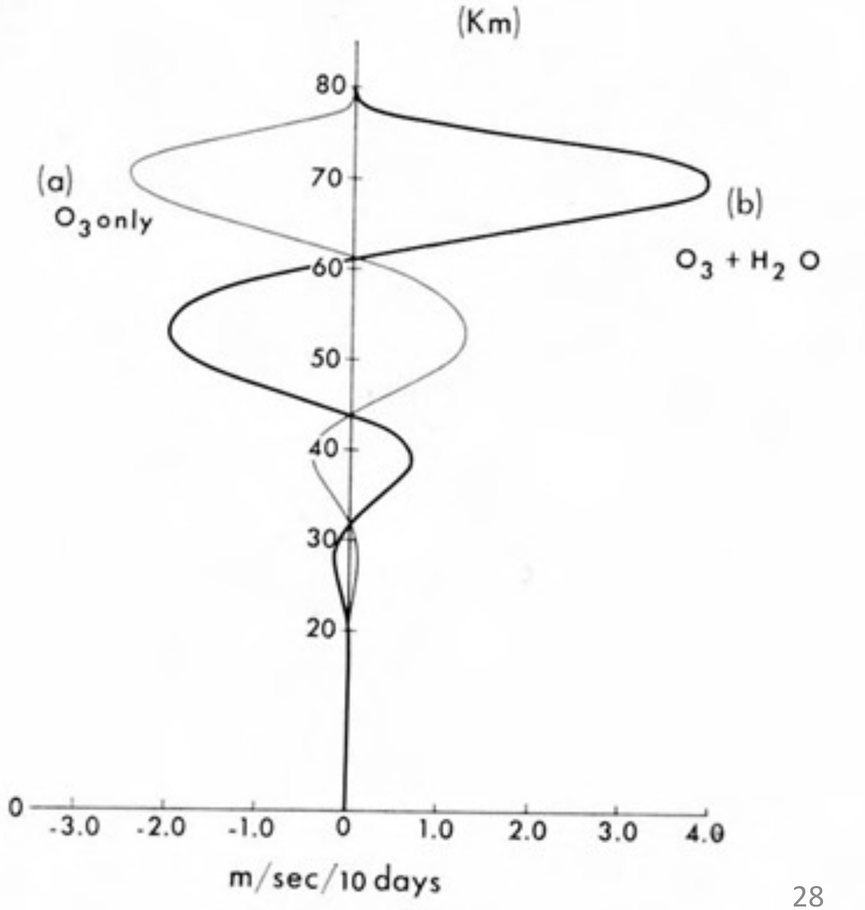
- Generalisation of EP theorems
  - cf Andrews & McIntyre 1976,1978]

$$\overline{w'p'} = -\rho_0(\bar{u} - c)\overline{u'w'}$$

- $\overline{w'p'} \sim$ Vertical energy flux
- A wave with eastward *c* (relative to $\bar{u}$) carries eastward momentum upwards

$$-\rho_0 \frac{\partial \bar{u}}{\partial t} = \frac{\partial}{\partial z}(\rho_0 \overline{u'w'}) = -\frac{\kappa \rho_0}{(\bar{u} - c)} \overline{D'J'}$$

- A thermally excited, propagating IGW will carry energy away from the source region
- Outside source region $\frac{\partial}{\partial z}(\rho_0 \overline{u'w'}) = 0$ but momentum has to come from somewhere – i.e. from $\bar{u}$!
- So within the source region, $\Longrightarrow$ eastward *c* wave produces westward acceleration of $\bar{u}$ (and vice-versa)



Simplified theoretical and numerical models: thermal tides

Stephen B. Fels

Richard Lindzen

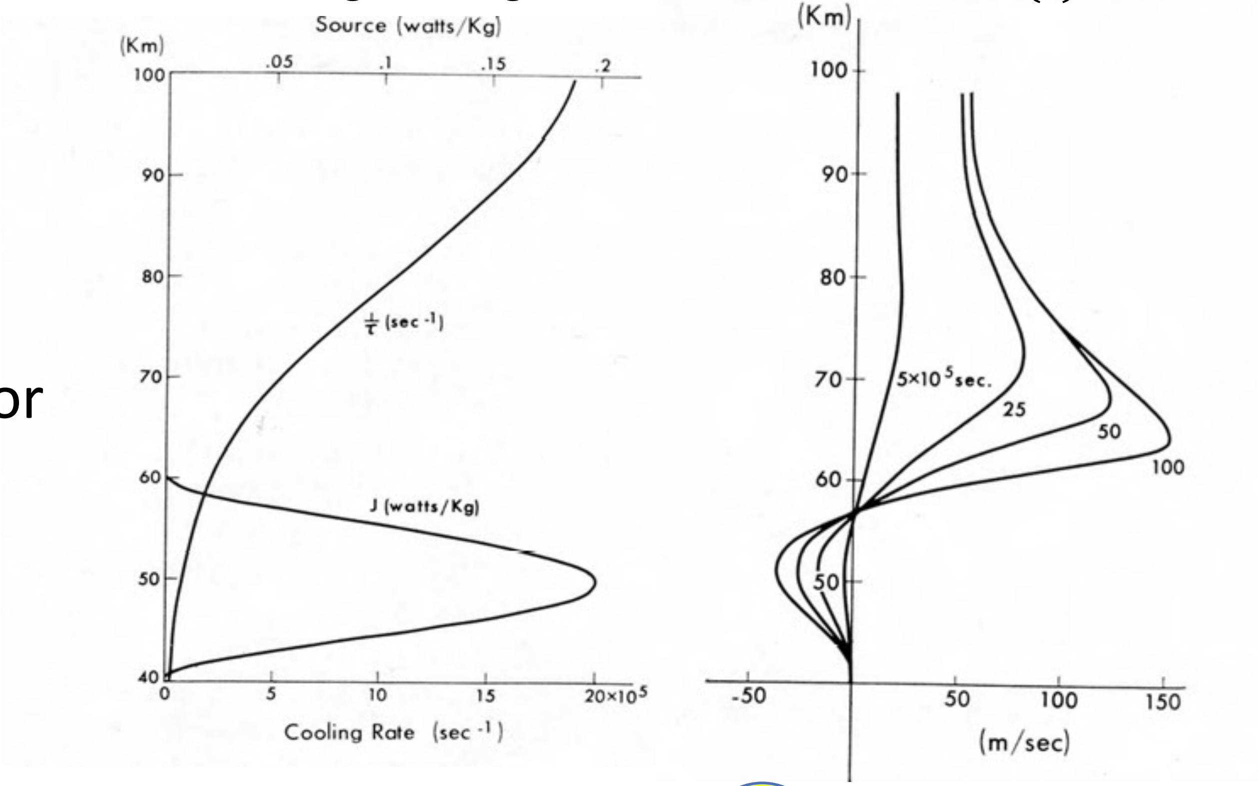
- For a finite thickness layer (depth H_h)

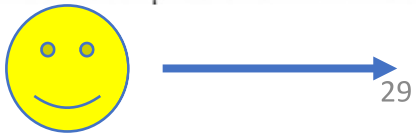
$$\left|\frac{\partial \bar{u}}{\partial t}\right| \approx \left(\frac{\mathcal{P}\lambda_v}{2H_h}\right)^2$$

  - Where $\mathcal{P}$ is power input per m^2 and λ_v is vertical wavelength
  - Sign of acceleration depends on heating or cooling
  - Example inspired by Venus conditions

Heating/cooling rate

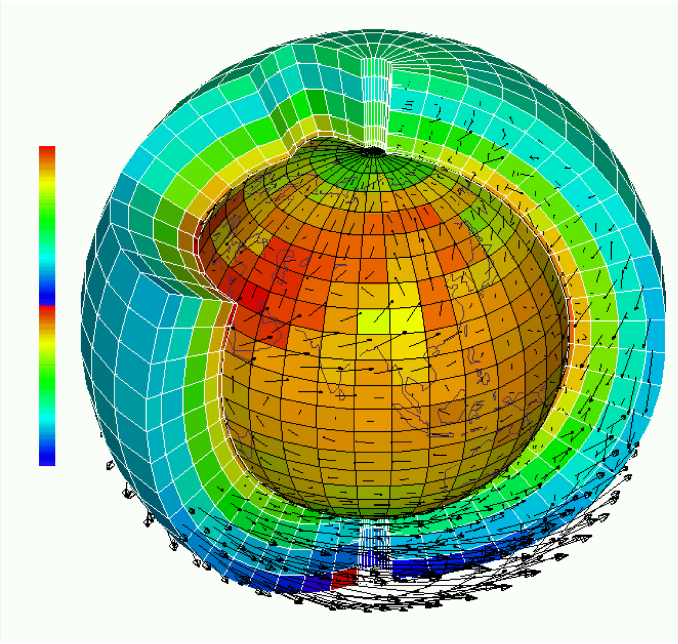
$\bar{u}(t)$

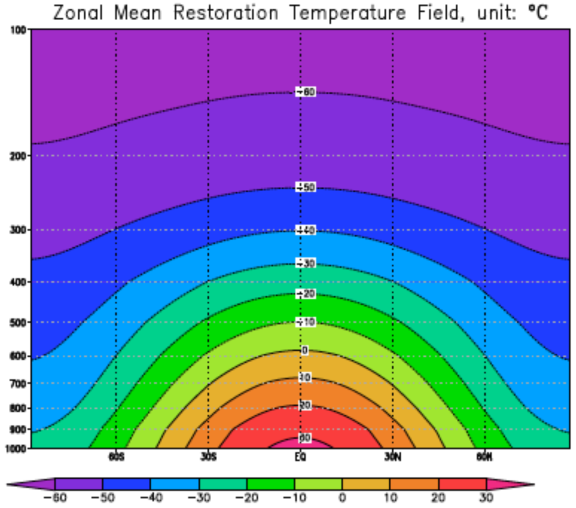




Simplified/intermediate complexity GCMs

- Basic Primitive Equation dynamical core
  - Numerically solves full (shallow) equations for conservation of momentum, energy and mass
  - Moderate resolution [7.5°x7.5° – 1°x1°], 10-25 levels
  - Simple diabatic forcing
    - Linear relaxation to prescribed $T_{eq}(\varphi, z)$; $\frac{\partial T}{\partial t} = \frac{(T_{eq}(\varphi,z)-T)}{\tau_{rad}}$ OR
    - Semi-gray (2-band) radiative transfer
  - Dry convective adjustment
  - WEAK diffusion or hyperdiffusion
- Simplified boundary conditions
  - Thermally insulating surface
  - Globally uniform - no topography
  - Linear frictional drag at surface [Time constant τ_{fr}]



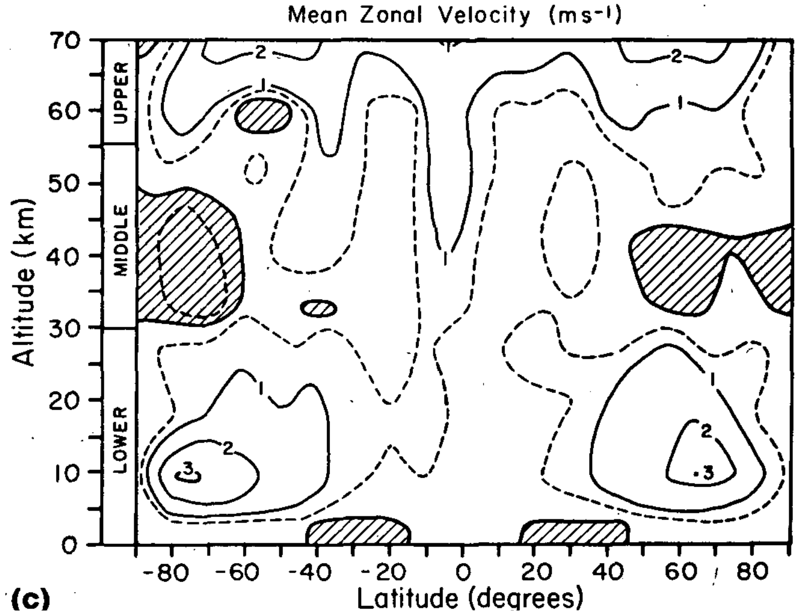


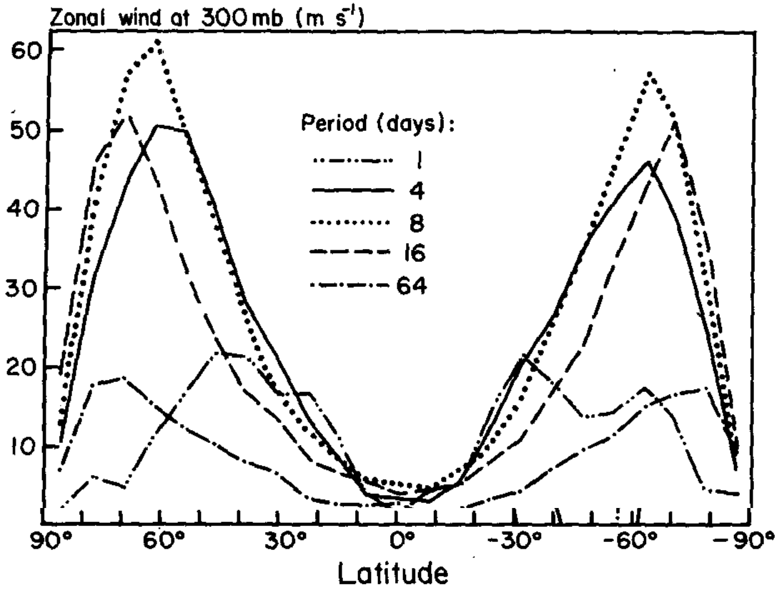
[Cf Earth in Perpetual equinox]

[Wang et al. 2018]

Simplified/intermediate complexity GCMs

- A long-established tool for atmospheric modellers
  - Since the early 1970s for other planets
- For Venus?
  - Early models produced only very weak super-rotation under Venus-like conditions (deep atmosphere, slow rotation)
  - E.g. Rossow [1980]
  - DelGenio & Suozzo [1987]
- Possible reasons?
  - Too diffusive?
  - Vertical diffusion and/or convective momentum transport too strong?
  - No diurnal cycle….?
  - AM non-conservation by numerical schemes….?





Simplified/intermediate complexity GCMs: A cautionary tale

- Venus SGCM intercomparison [Lebonnois et al. 2014]
- 6 different models with total of 8 different dynamical cores
  - Includes spectral and finite-difference cores
- Similar (Venus-like) conditions
  - Same T_{eq} fields and τ_{rad}
  - Similar resolutions (not identical)
  - Linear surface drag

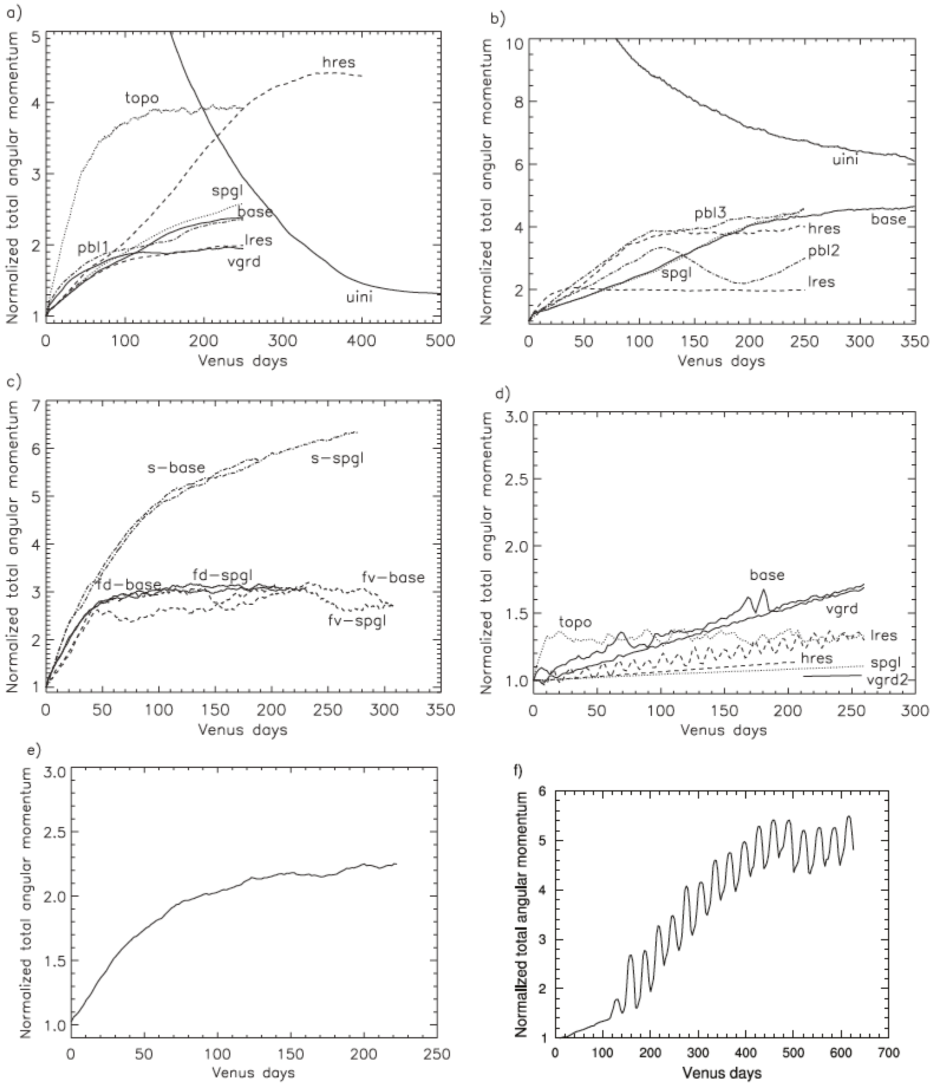
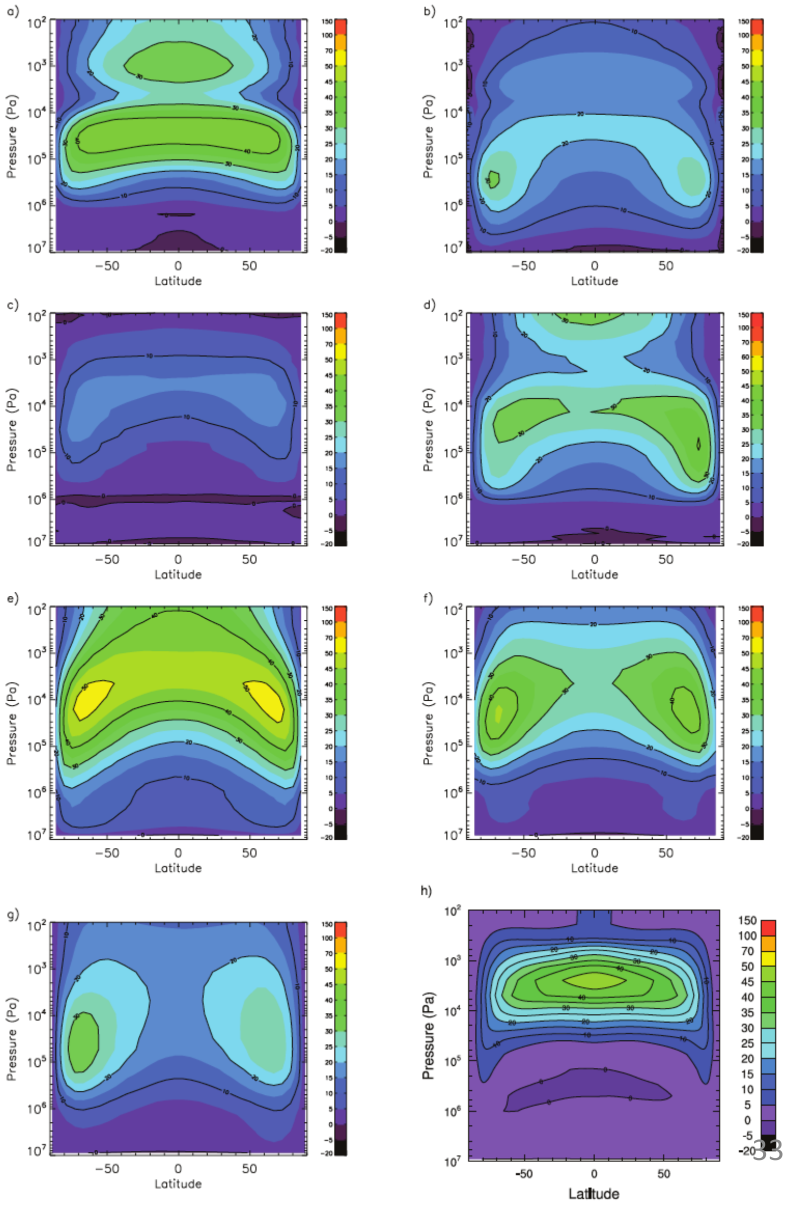


Figure 7.1: Evolution of the total atmospheric angular momentum, normalized to its initial value (with atmosphere at rest), for all the simulations. (a) CCSR, (b) LMD, (c) LR10, (d) OU, (e) OX, (f) UCLA.

Simplified/intermediate complexity GCMs: A cautionary tale

- Venus SGCM intercomparison [Lebonnois et al. 2014]
- 6 different models with total of 8 different dynamical cores
  - Includes spectral and finite-difference cores
- Very different results!
  - LH = spectral models
  - RH = FD models
  - Last line = finite volume schemes

$\bar{u}$



Simplified/intermediate complexity GCMs: A cautionary tale

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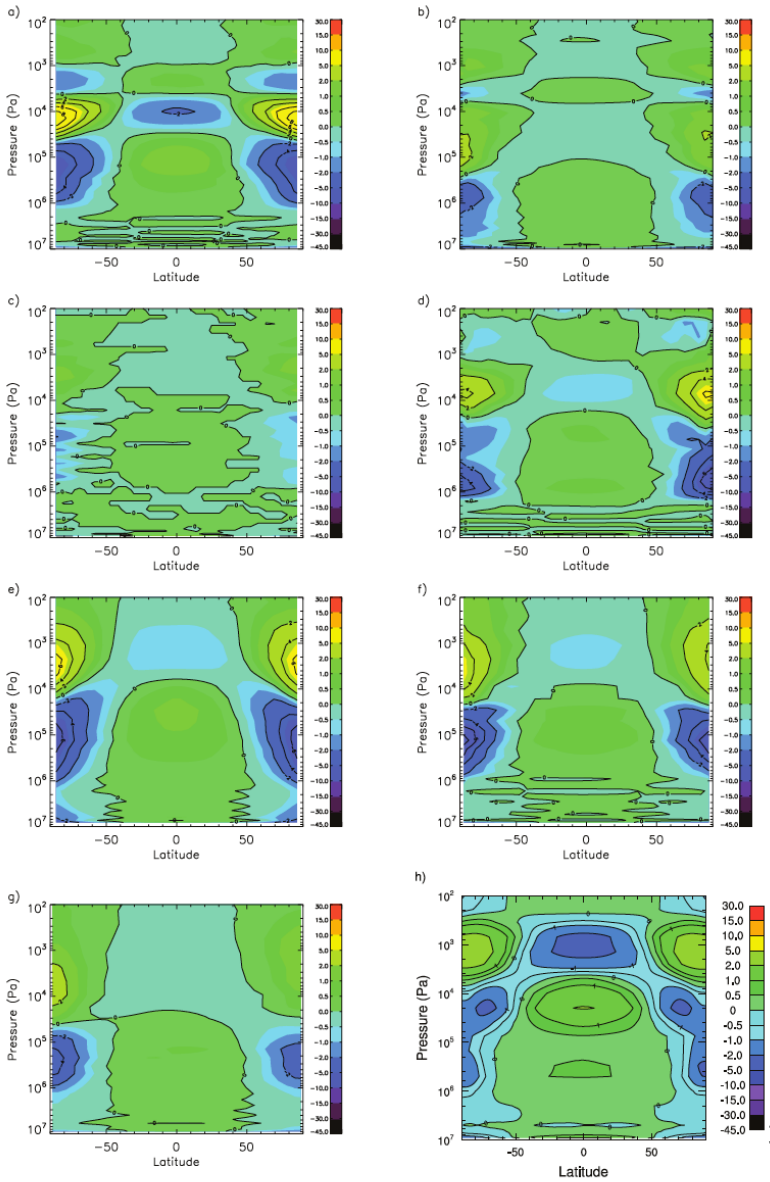
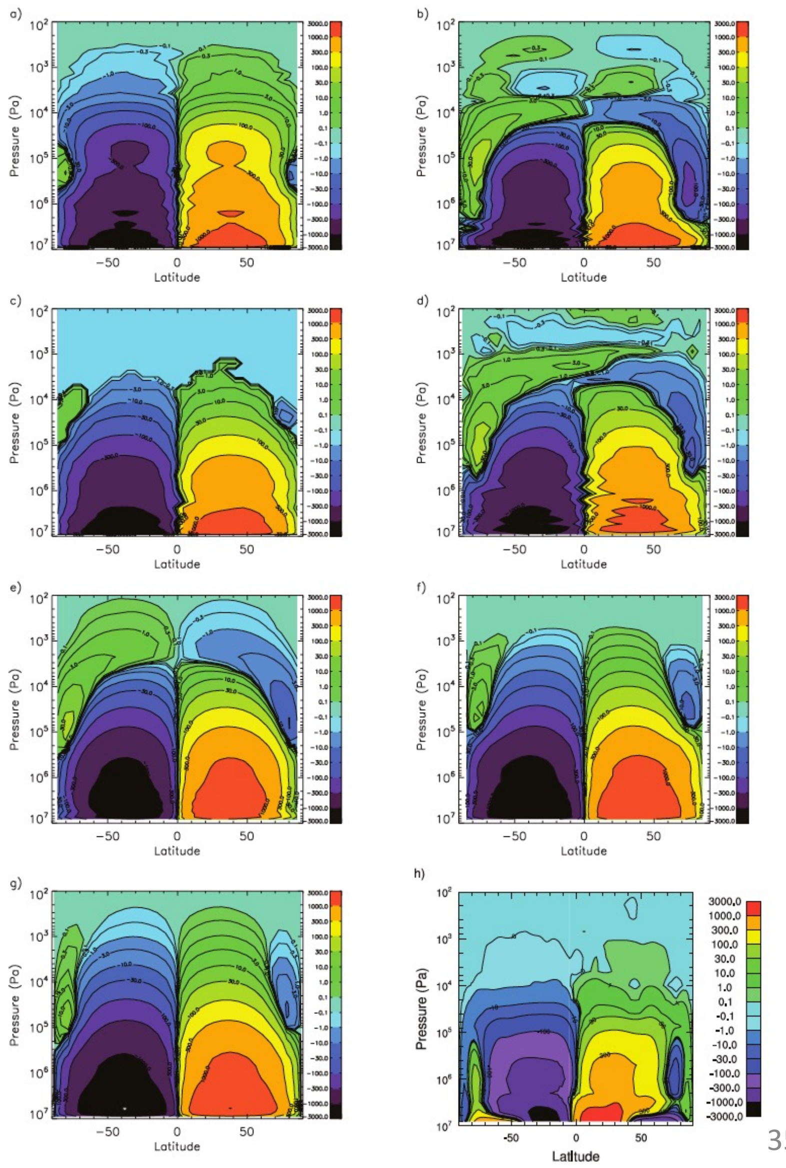
$$\bar{T} - T_0(z)$$



Simplified/intermediate complexity GCMs: A cautionary tale

$\overline{\Psi}$



- Venus SGCM intercomparison [Lebonnois et al. 2014]
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Simplified/intermediate complexity GCMs: A cautionary tale

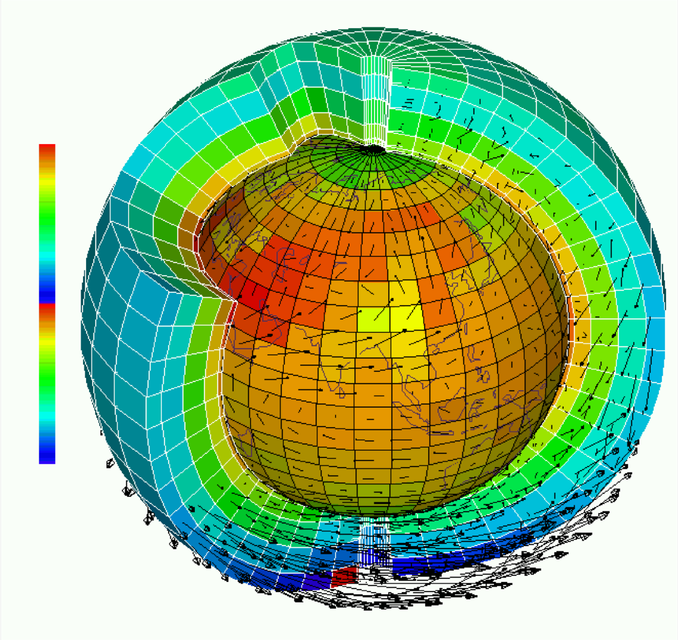
Conclusions

- Numerical models are not perfect physical analogues of atmospheric circulation – especially for slow rotators
- Results may be sensitive to resolution, choice of dynamical core, numerical advection schemes and other factors
- Deep atmospheres surrounding slowly rotating planets are especially challenging because of sensitivity to weak sources or sinks of AM, implicit numerical diffusion etc.
- Results may need to be treated with caution and/or tested for robustness against changes in resolution and other factors.

Simplified/intermediate complexity GCMs: Parameter sweeps and scaling

- Although particular simulations may not be perfectly accurate, they may be useful to explore trends in atmospheric circulation at different values of key parameters
- Explore possible scaling laws and other trends in heat and angular momentum transfer e.g. as a function of planetary rotation rate and related parameters
- Several recent studies have attempted this. Here we look at some results using the University of Hamburg PUMA and University of Exeter ISCA models
  - Wang et al. [2018 QJRMS]
  - Lewis et al. [2021 JAS]

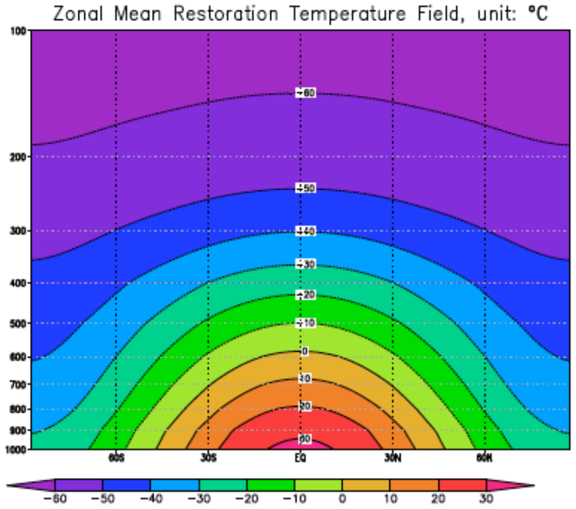
Exploring parameter space with a *simple* [3D] climate model



- Pseudo-spectral dynamical core - PUMA [Univ. of Hamburg] or ISCA [Univ. Exeter]
  - Solves hydrostatic Primitive Equations for conservation of momentum, energy and mass using spherical harmonics in horizontal, FD in vertical
  - T21-T170 [7.5°x7.5° – 1°x1°], 10-25 levels
- Flat surface (no topography)
- Simple radiative forcing
  - Linear relaxation to specified $T(\phi,z)$ OR
  - Semi-gray (2-band) radiative transfer
- Linear frictional drag at surface
  - Time constant τ_{fr}
- Vary Ω, τ_{rad} or τ_{fr}

$$\boxed{\frac{1}{256} \leq \frac{\Omega}{\Omega_E} \leq 8}$$

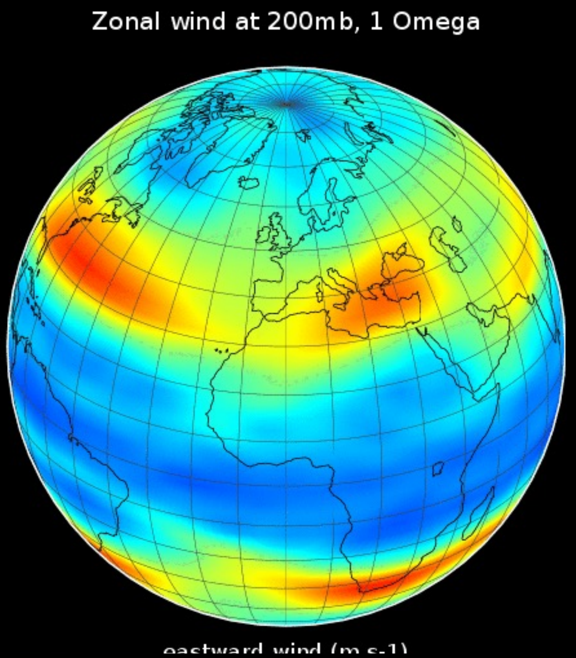
- No moisture or oceans

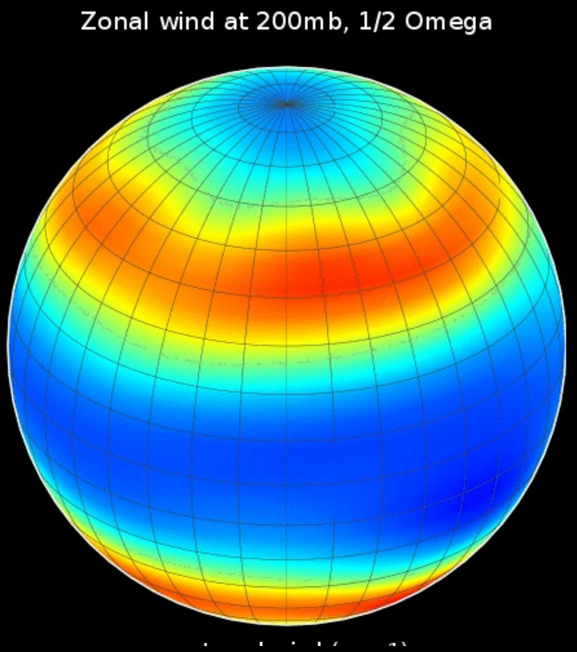


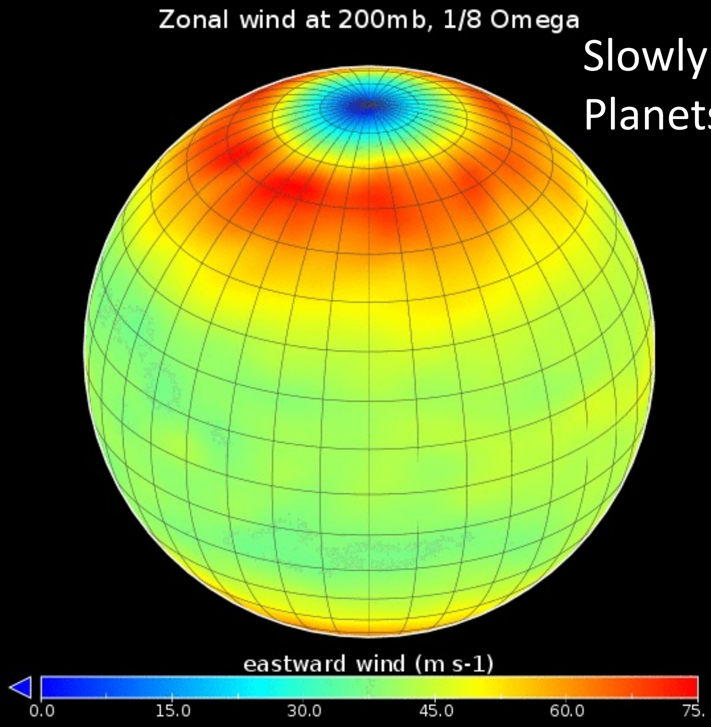
[Wang et al. 2018]

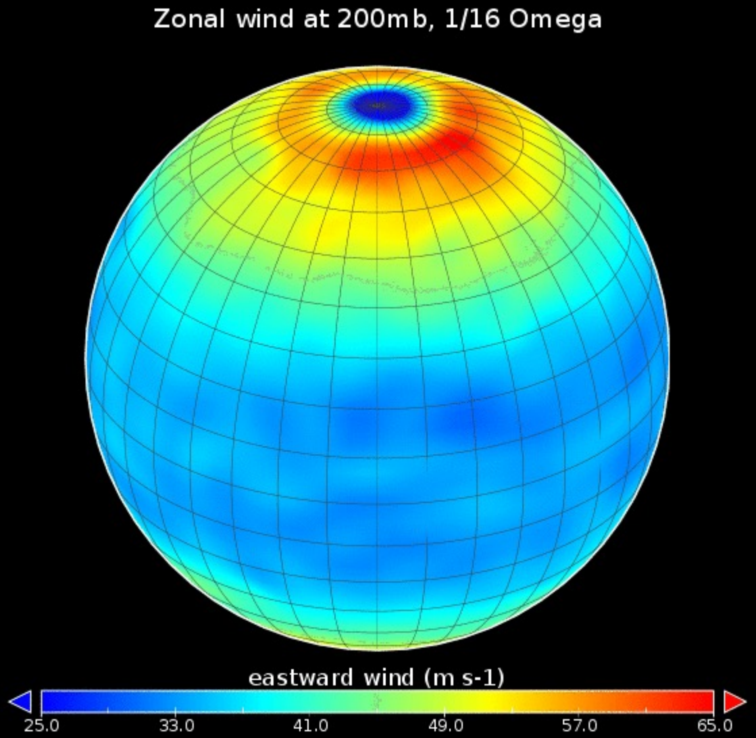
[Cf Earth in Perpetual equinox]

Slowly rotating Planets ($\Omega < \Omega_E$)

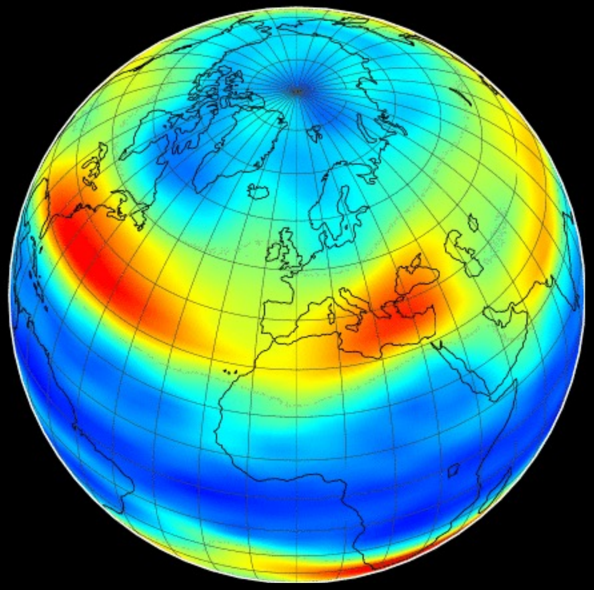




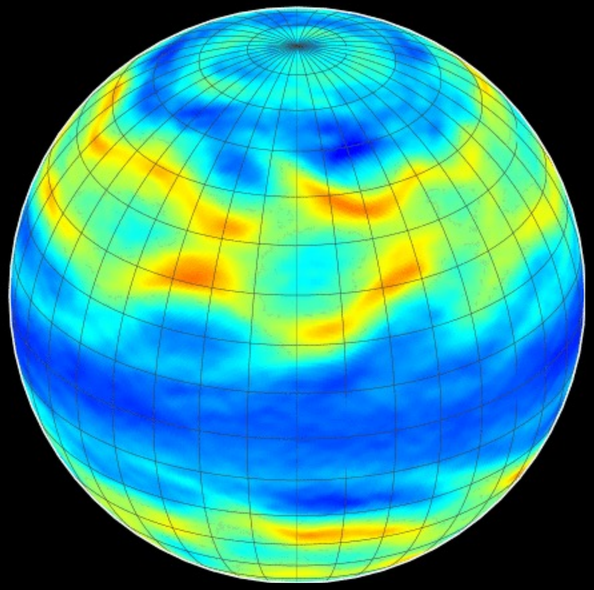




Zonal wind velocity at 200mb (Earth)

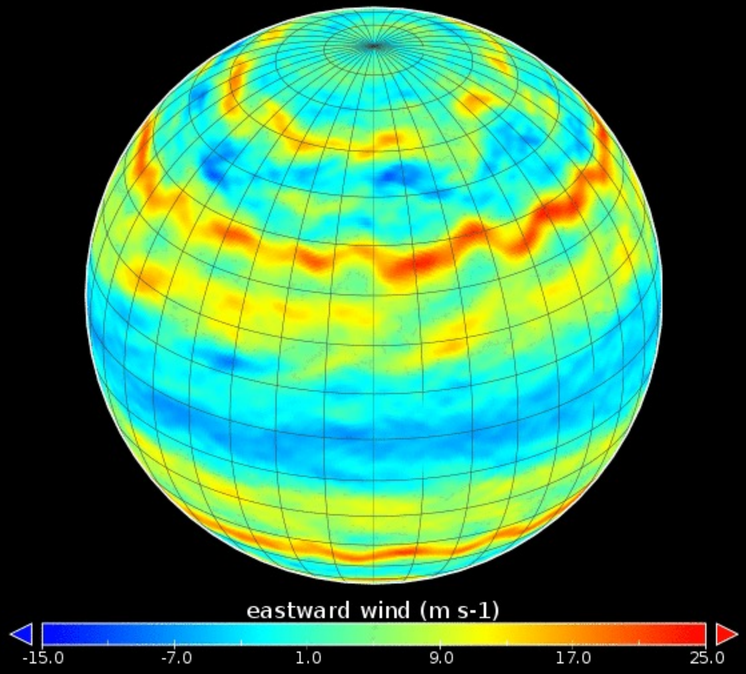


Zonal wind at 200mb, 2 Omega

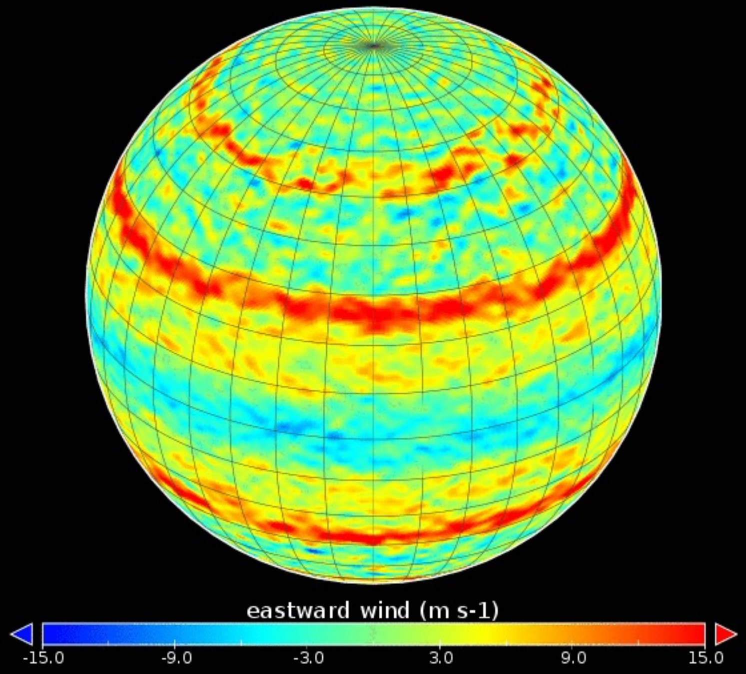


Rapidly Rotating Planets ($\Omega > \Omega_E$)

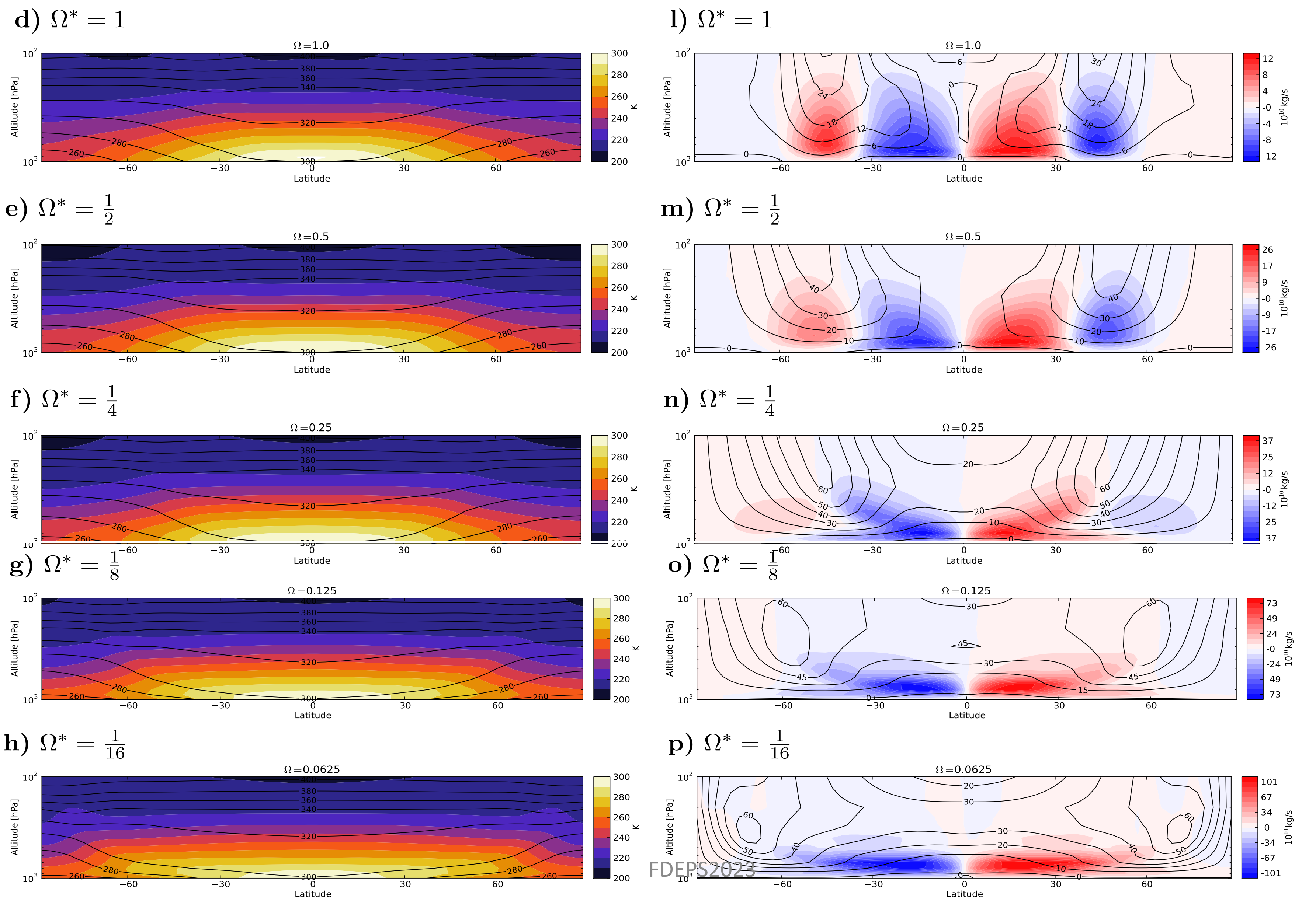
Zonal wind at 200mb, 4 Omega



Zonal wind at 200mb, 8 Omega



Trends in circulation structure: decreasing Ω



$$\Omega = \Omega_E$$

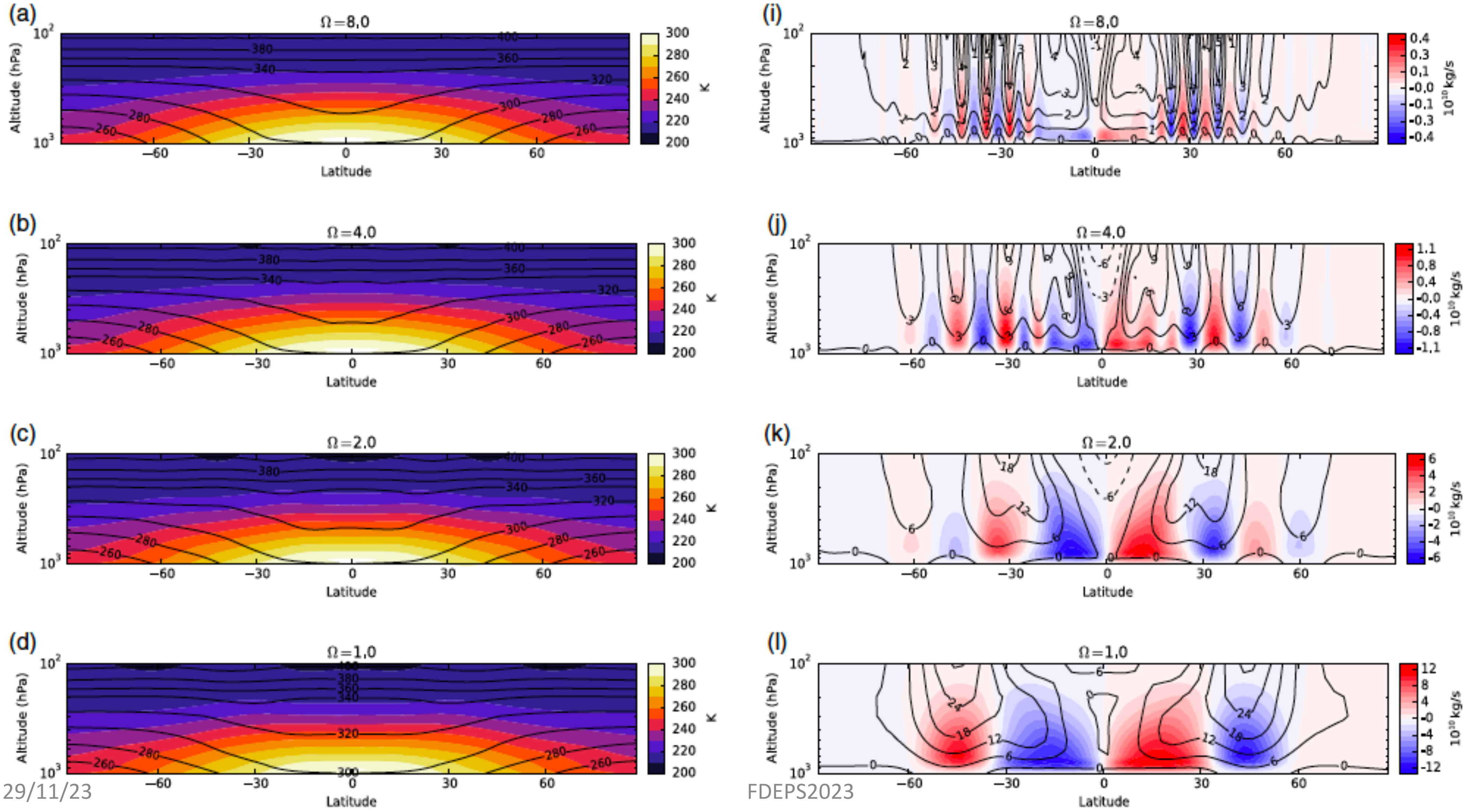
$$\Omega = \Omega_E/2$$

$$\Omega = \Omega_E/4$$

$$\Omega = \Omega_E/8$$

$$\Omega = \Omega_E/16$$

Trends in circulation structure: increasing Ω



$\Omega = 8\Omega_E$

$\Omega = 4\Omega_E$

$\Omega = 2\Omega_E$

$\Omega = \Omega_E$

Order according to key planetary parameters defining circulation regimes?

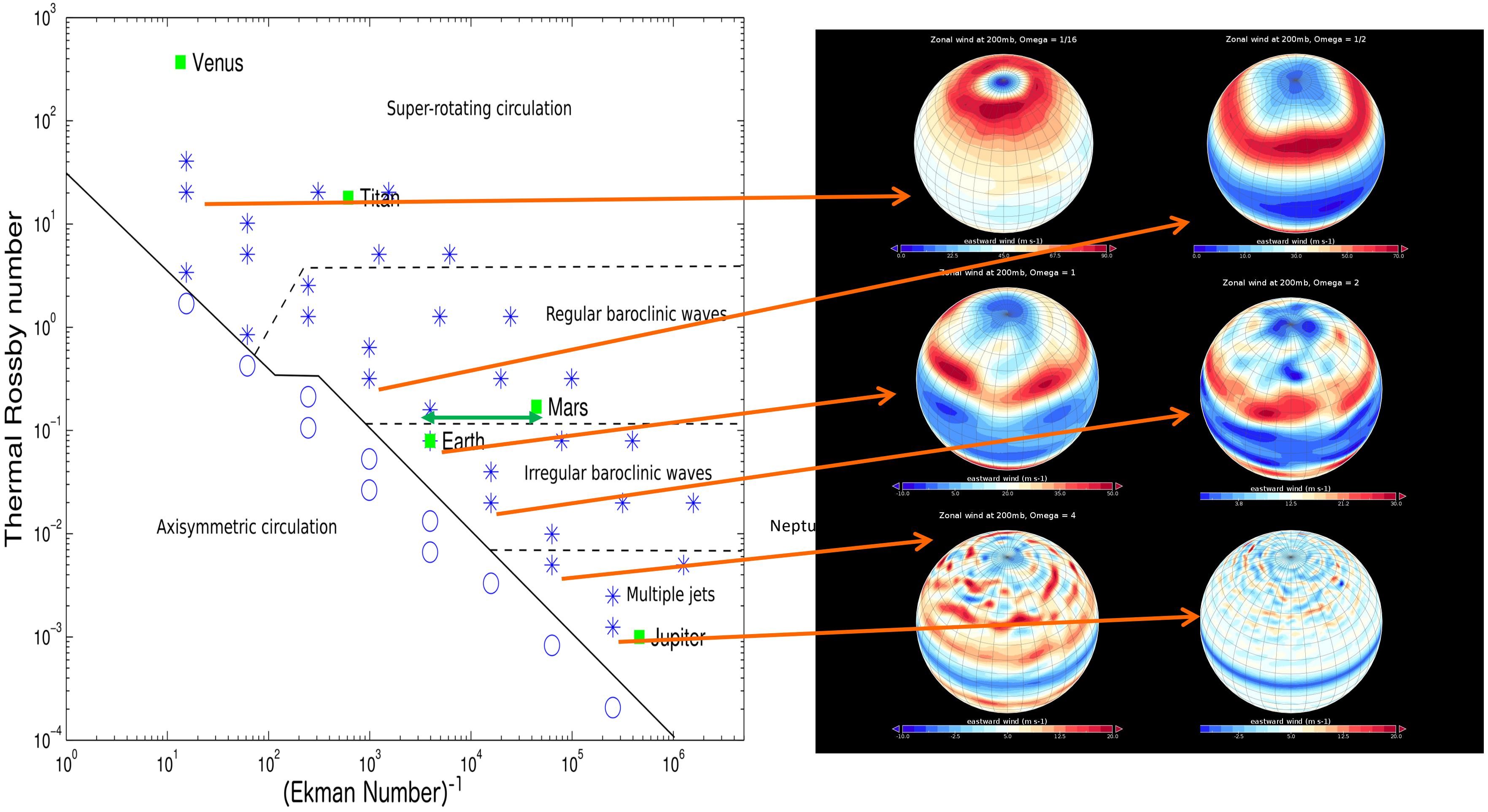
- Thermal Rossby [R.Hide] number *[-Ratio of forces]*
  - $\mathcal{R} = \frac{U_T}{\Omega L} \approx \frac{gH\Delta_h\theta}{\theta_0\Omega^2 a^2} = \frac{R\Delta_h\theta}{\Omega^2 a^2}$
    - $H = \frac{RT}{g}$; atmospheric scale height
- Burger number *[-Ratio of lengthscales]*
  - $\mathcal{B} = \frac{R\Delta_v\theta}{4\Omega^2 a^2} = \frac{L_d^2}{a^2}$
    - $L_d = \frac{NH}{f}$; Rossby deformation radius

Order according to key planetary parameters defining circulation regimes?

- Damping/dissipation parameters *[Ratios of timescales]*
  - Frictional Ekman number $E_{fr} = \left(\Omega\tau_{fr}\right)^{-2}$; [cf $E = (\Omega\tau_{visc})^{-2}$]
    - $\tau_{fr} = \tau_{drag}\left(\frac{H}{h_{BL}}\right)$; ["spindown timescale"; Valdes & Hoskins 1988]
  - Radiative damping parameter $\mathcal{A}_a = \frac{2\pi}{\Omega\tau_{rad}} = \frac{\tau_{rot}}{\tau_{rad}}$
    - Radiative timescale $\tau_{rad} \approx \frac{c_p p_s}{\sigma g T_{eff}^3(2-\epsilon)}$; [transmissivity ϵ]
  - OR "Taylor numbers", $\mathfrak{I}_{fr,rad} \sim \left(E_{fr}^{-2}, \mathcal{A}_a^{-4}\right)$
  - Others…….?

Schematic regime diagram
[w.r.t. *dimensionless* parameters]

$\mathcal{R} \propto \Delta\theta_{EP}/[\Omega R]^2$



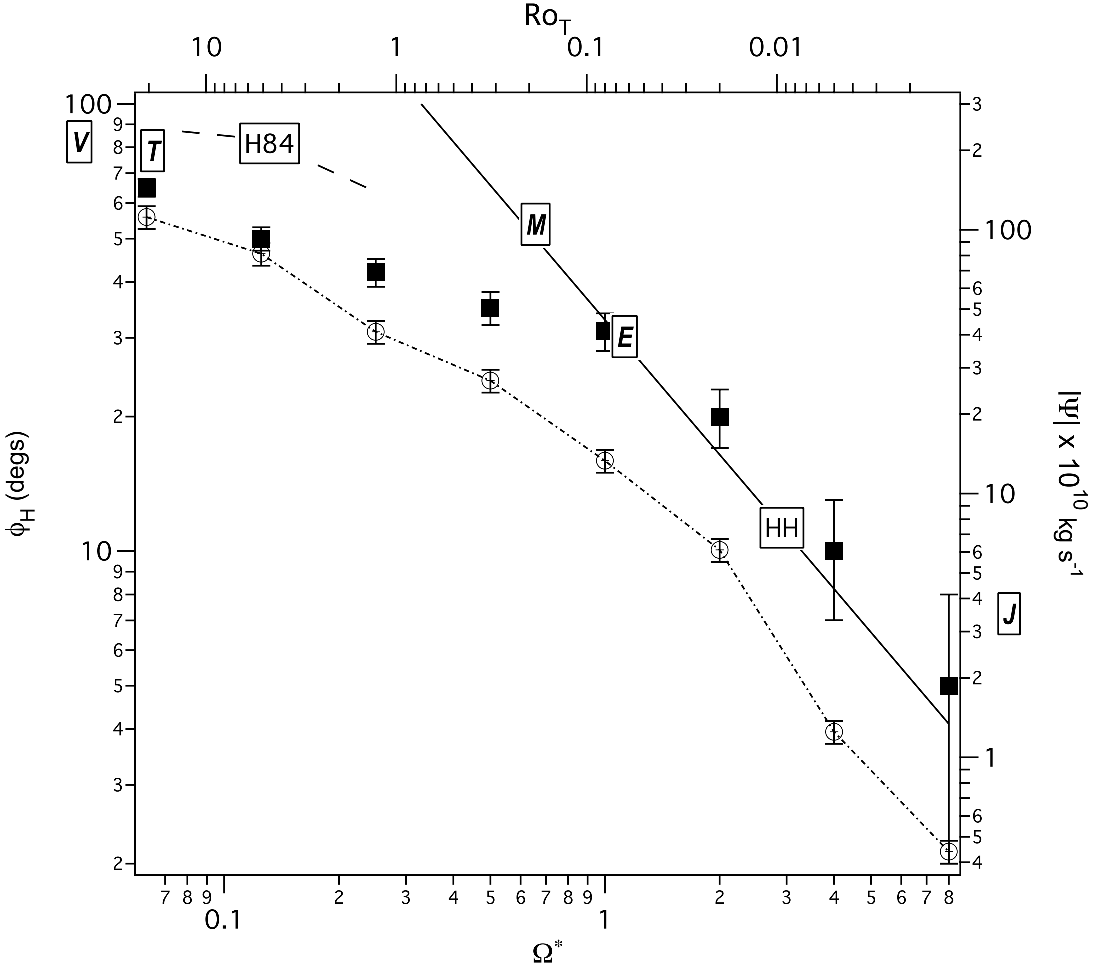
$\propto (\Omega\tau_f)^2$

Planetary parameters

Ω/Ω^*	Θ	N_J	$4\Omega^2\tau_R^2$	Cf
1/16	20	0.04	62	Titan
1/8	5	0.07	247	
1/4	1.3	0.14	992	
1/2	0.32	0.28	4000	Mars[?]
1	0.08	1.57	1.5e4	Earth
2	0.02	3.1	6.3e4	Uranus & Neptune?
4	0.005	6.3	2.5e5	
8	0.001	14.5	1.0e6	Saturn & Jupiter

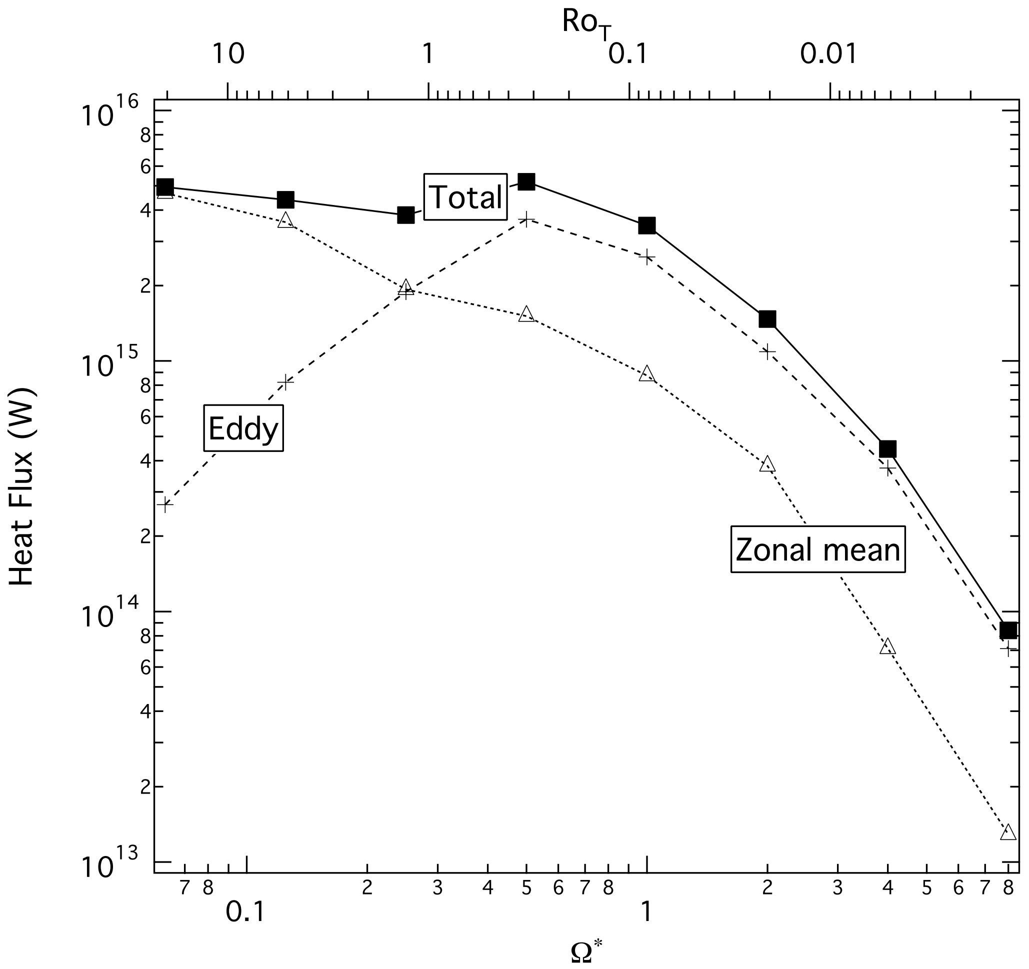
Simplified/intermediate complexity GCMs: Parameter sweeps and scaling

- Hadley circulation width ($|\varphi_H|$)
- and strength ($|\overline{\Psi}|$)
  - Width ϕ_H scales with $\mathcal{R}$, much as predicted by HH80
    - $\sim\mathcal{R}^{1/2}$ for small $\mathcal{R}$
    - ~whole planet for large $\mathcal{R}$
  - Strength $|\varphi_H|$ also scales with $\mathcal{R}$
    - $\sim\mathcal{R}^{2/5} - \mathcal{R}^{3/5}$ for $\mathcal{R} \gtrsim 1$



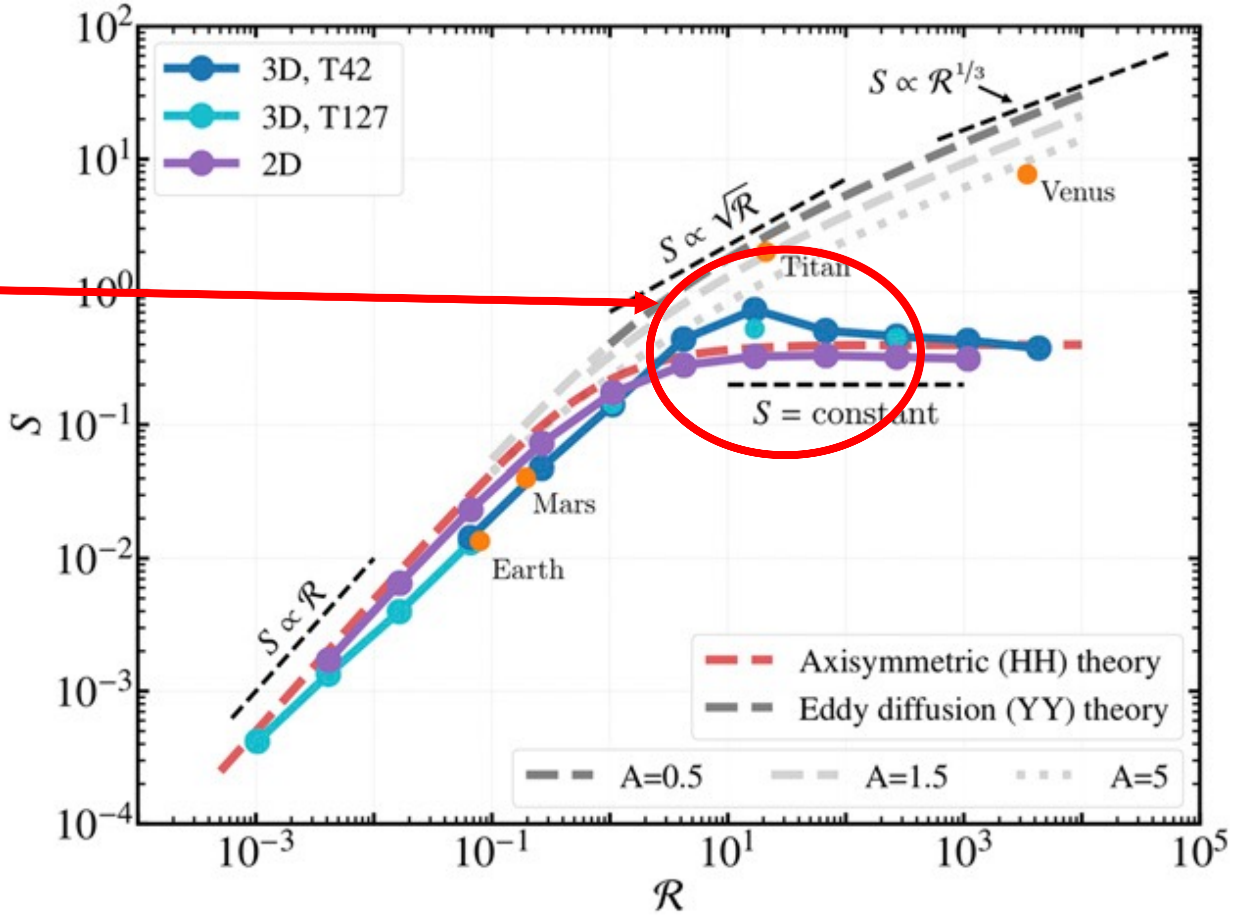
Simplified/intermediate complexity GCMs: Parameter sweeps and scaling

- Peak equator-pole heat transport
  - Total heat transport ~ constant for $\mathcal{R} > 1/2$ then decreases for smaller $\mathcal{R}$
  - Zonally symmetric overturning contribution decreases with increasing Ω^*
    - $\sim\mathcal{R}^{-1}$ for small $\mathcal{R}$
    - Dominant for $\mathcal{R} > 1$
    - Leads to weak horizontal T gradients
  - Eddy contribution peaks at $\mathcal{R} \approx 0.3$
    - ~negligible for $\mathcal{R} > 10$
    - Dominates for $\mathcal{R} < 0.1$
      - Baroclinic instabilities



Simplified/intermediate complexity GCMs: Parameter sweeps and scaling

- Global super-rotation S
- 3D simulations stay close to the HH80 theory except around $\mathcal{R}\sim 10-100$
- $\bar{u}$ in gradient wind balance but doesn't quite reach full cyclostrophic conditions

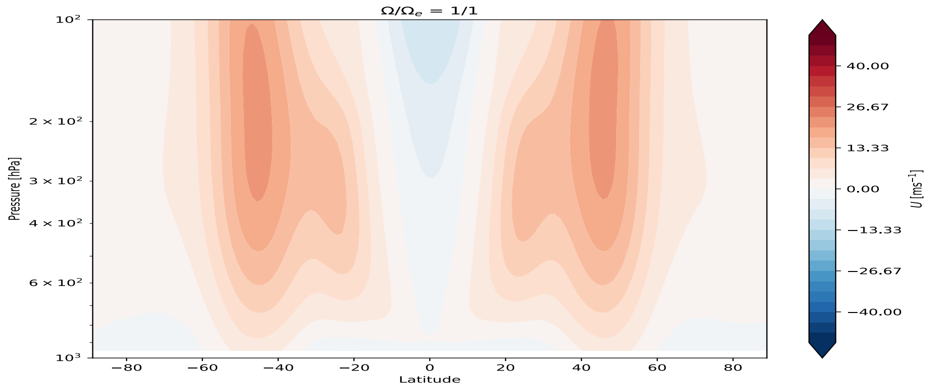


Zonal wind fields (Isca)

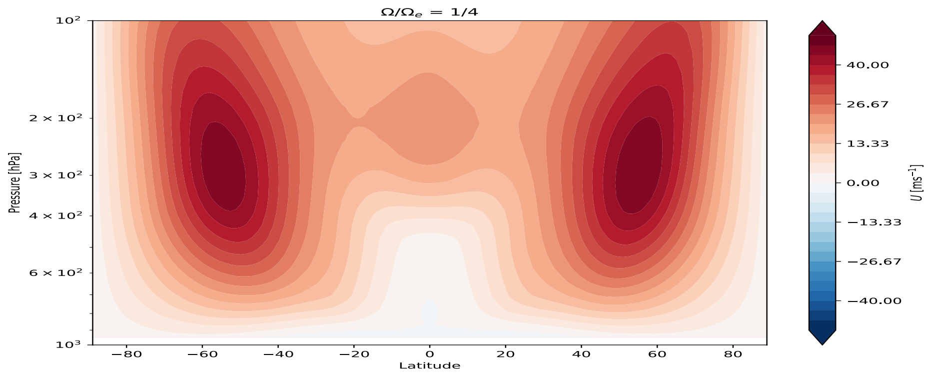
- Earth-sized planet
- Held-Suarez relaxation forcing
- p_S = 1 bar

Ω_*

1

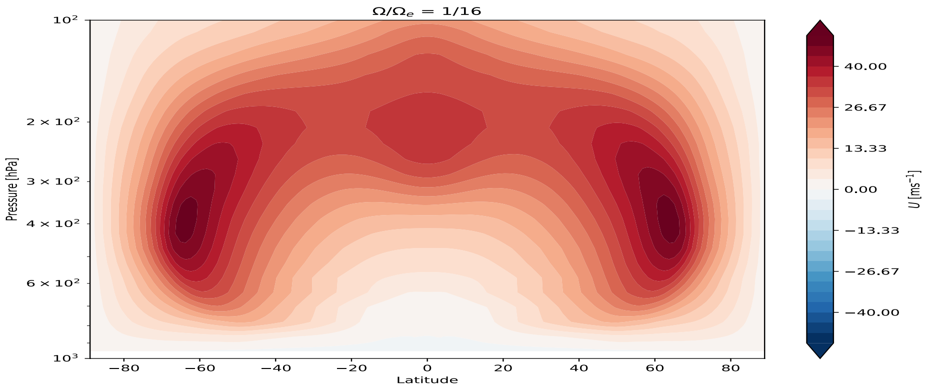


1/4



Ω_*

1/16



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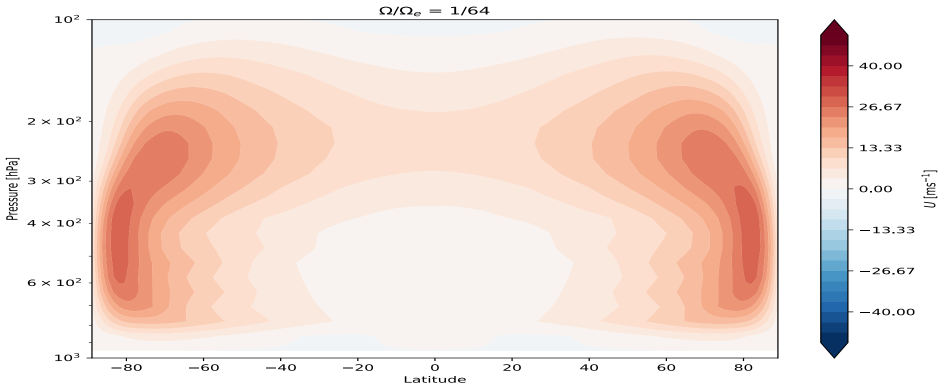

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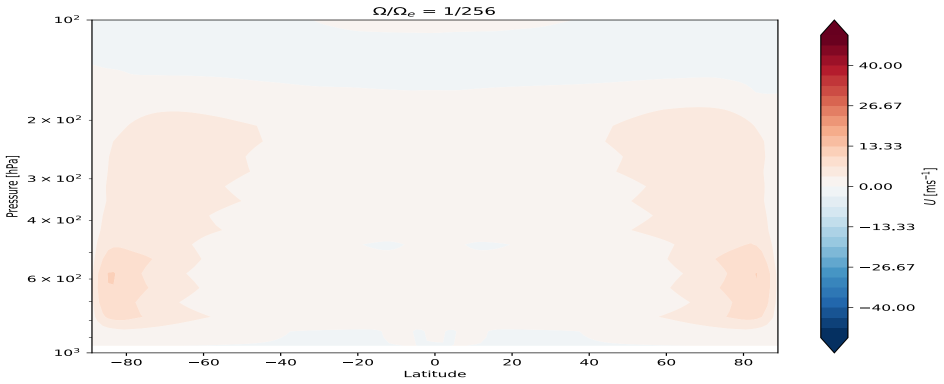
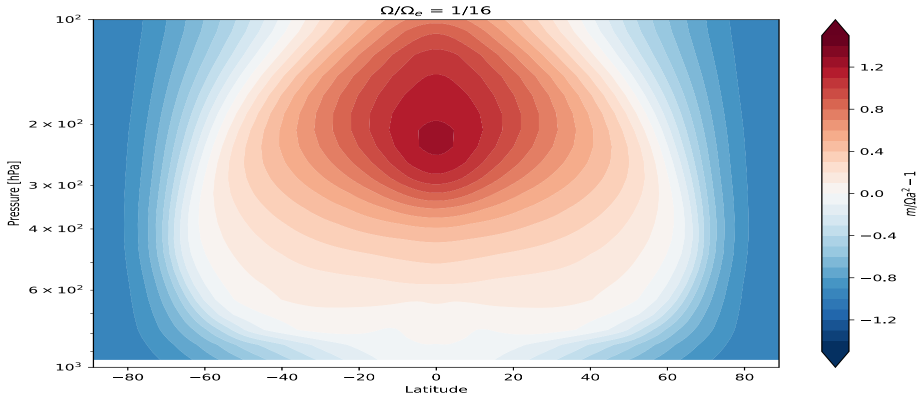


Local super-rotation fields s_{local} (Isca)

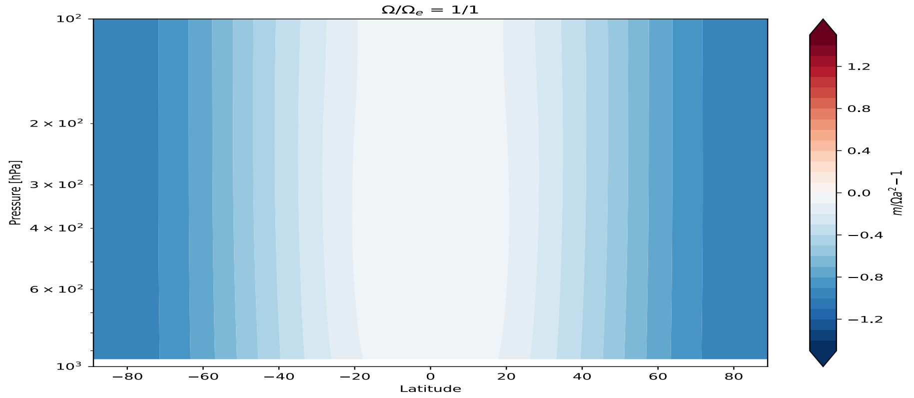


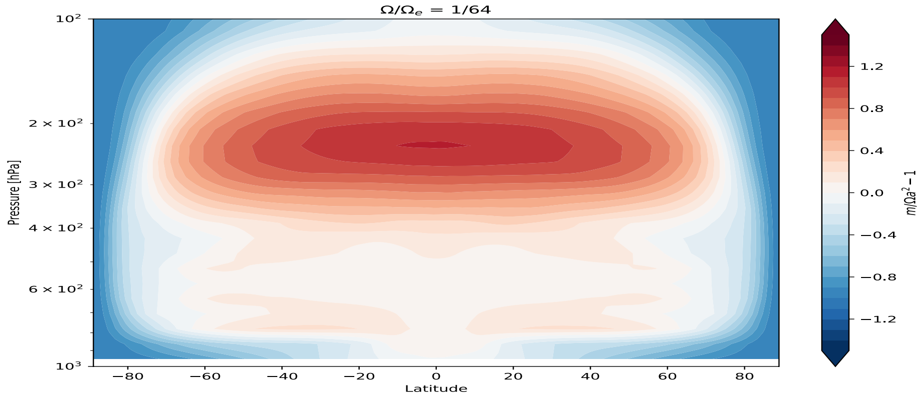
Ω_*

1/16

Ω_*

1





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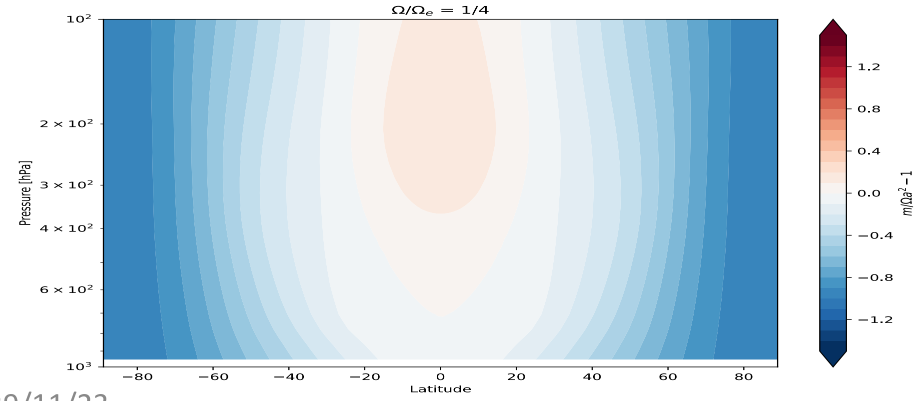
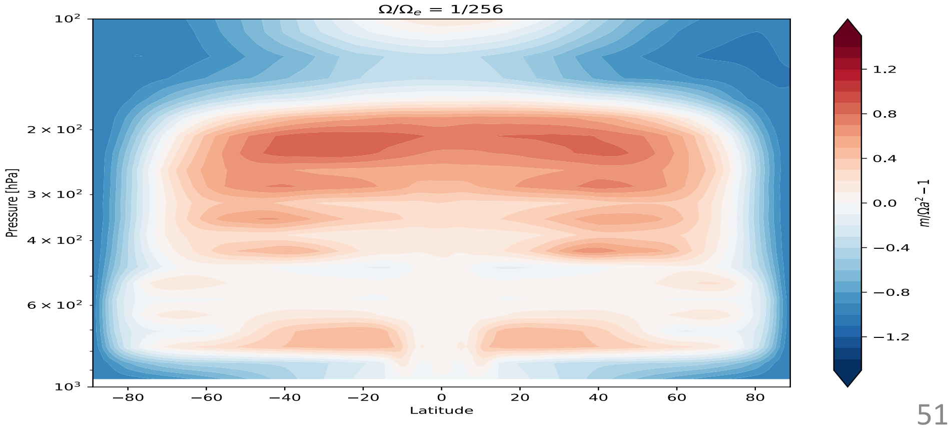
1/4





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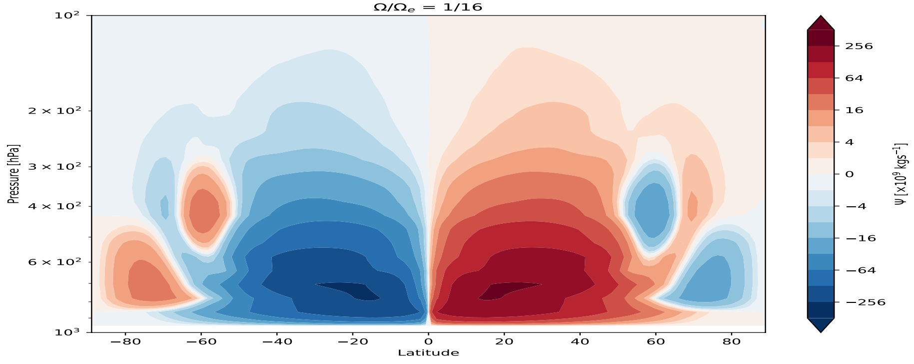
Mean meridional mass streamfunction (Isca)

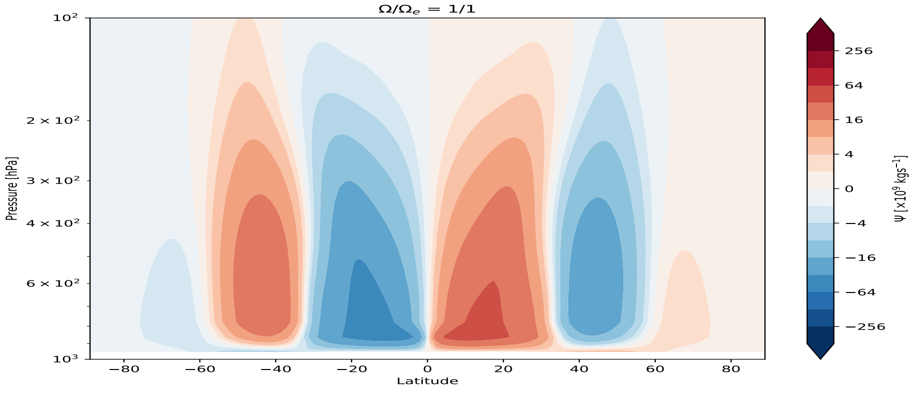
Ω*

1/16



Ω*

1



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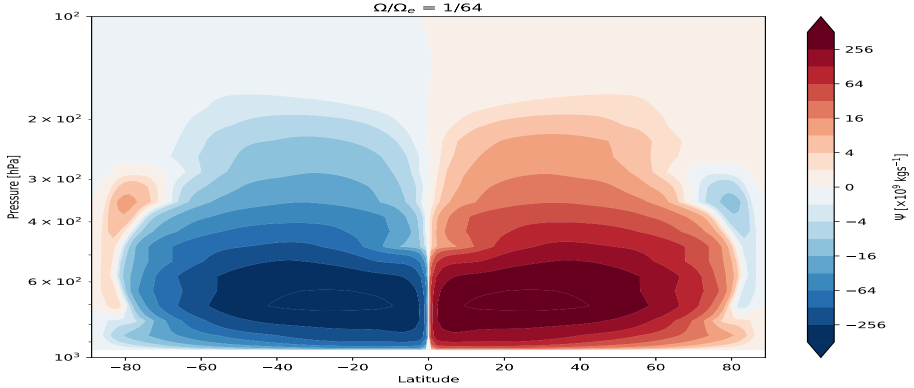

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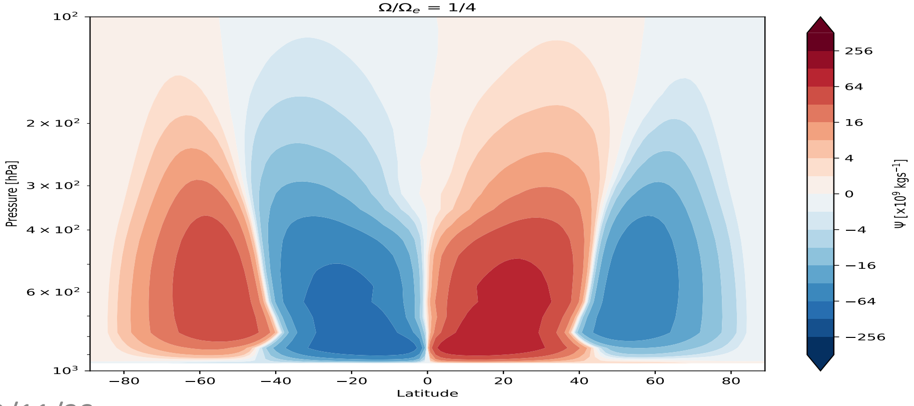

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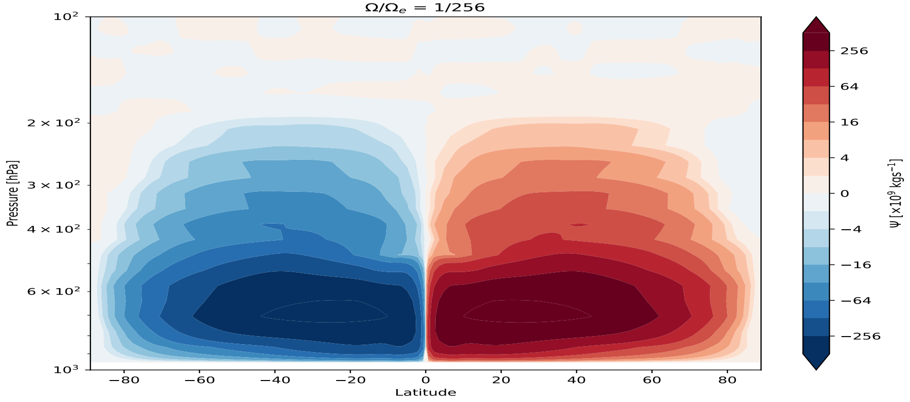
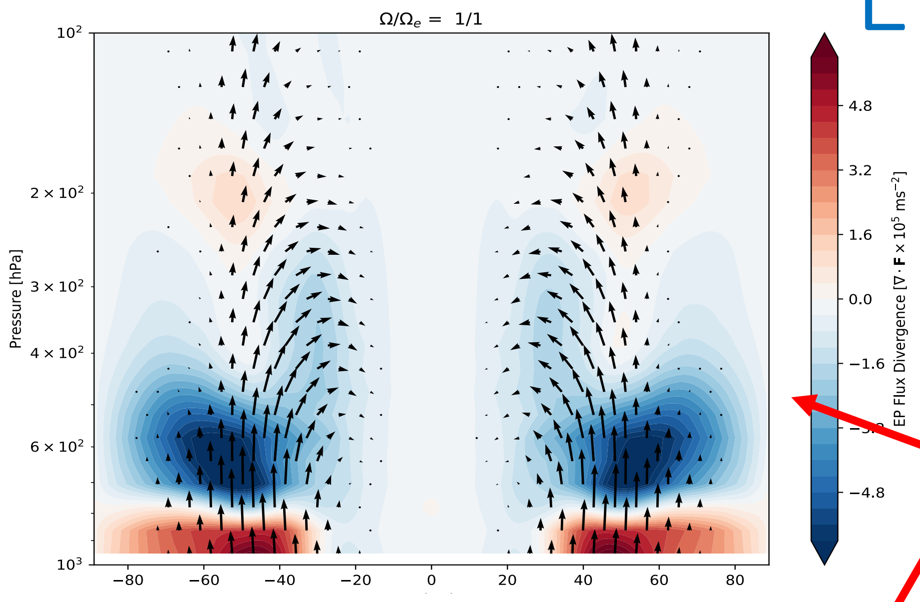
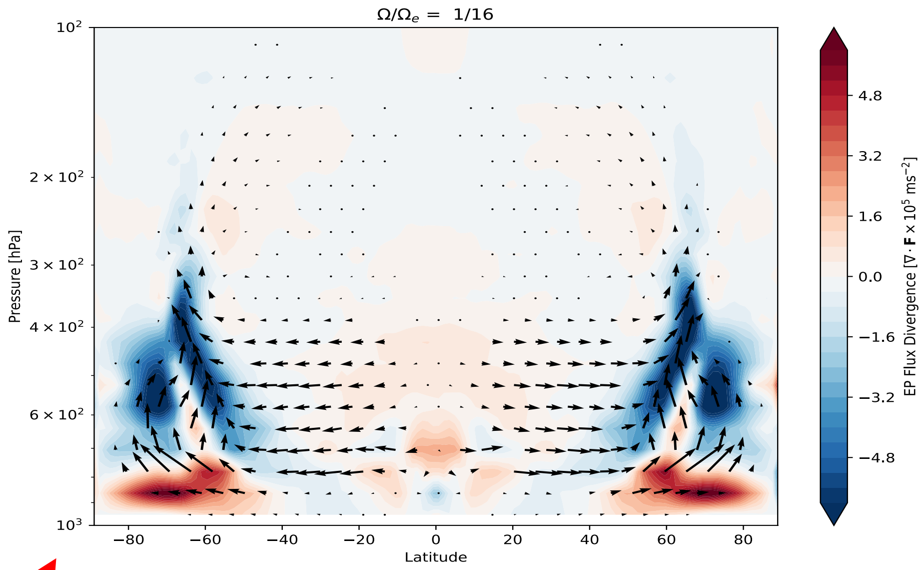


EP Fluxes(Isca)



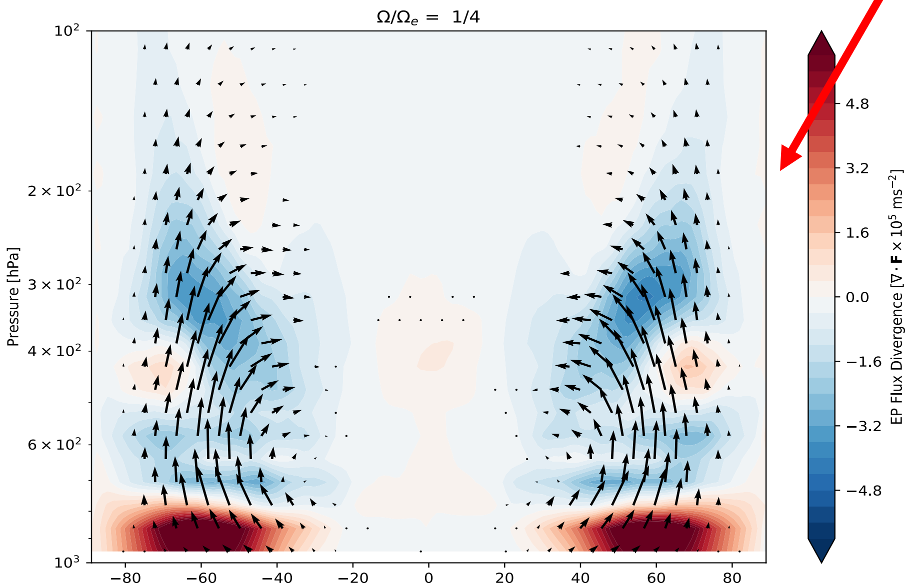


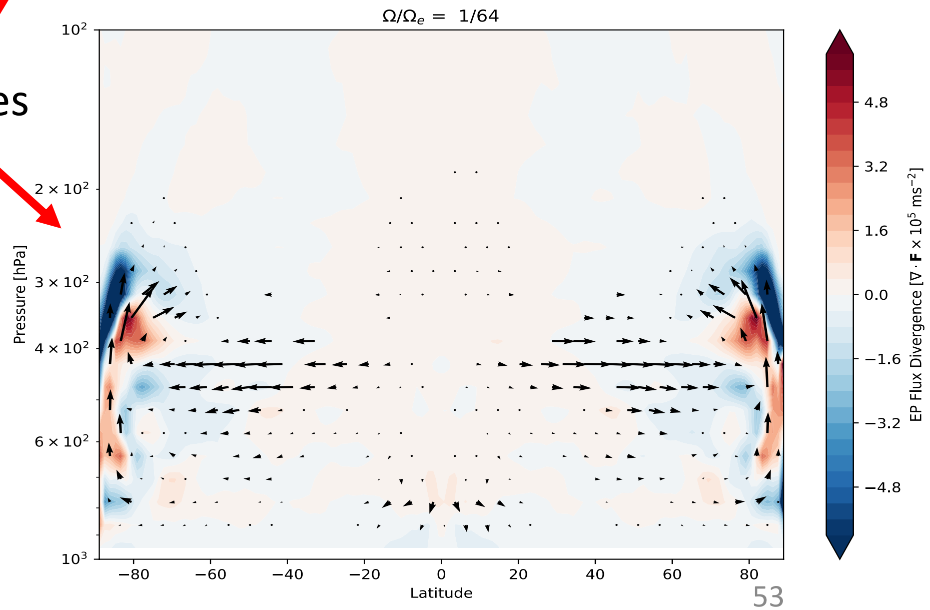
Ω_*	Ω_*
1	1/16

Dominated by
Baroclinic (Rossby) waves
at mid-latitudes

Dominated by
Equatorial Kelvin/Rossby waves

1/4	1/64





Simplified/intermediate complexity GCMs: Parameter sweeps and scaling

- Some models are more successful at reproducing strong super-rotation at Venus-like rotation rates
  - E.g. Yamamoto & Takahashi [2003]
  - Strong $\overline{u'v'}$ equatorward momentum fluxes maintain ~100 m s^{-1} equatorial winds
  - Dominant wave modes include Rossby, Kelvin and gravity waves
- Role of equatorial Kelvin waves?

2 - 2 YAMAMOTO AND TAKAHASHI: VENUSIAN SUPERROTATION

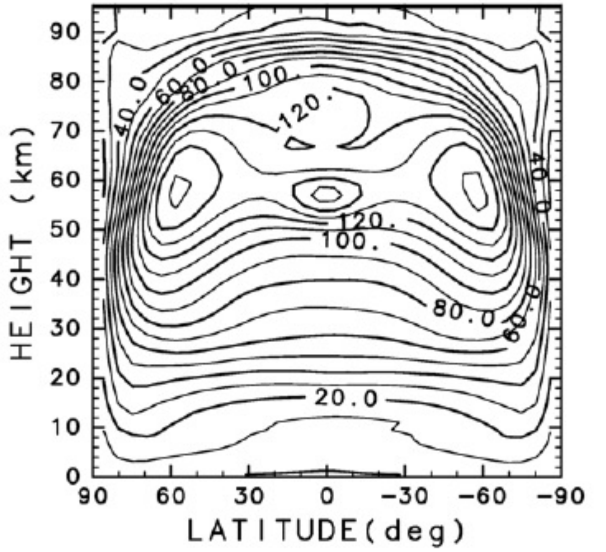


Figure 1. Latitude–height cross section of longitudinally averaged zonal flow (m s^{-1}).

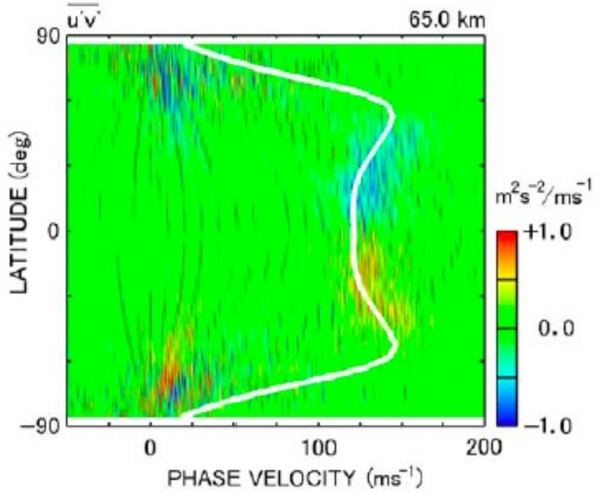


Figure 2. Phase-velocity–latitude cross section of spectrum of $\overline{u'v'}$ at 65 km altitude. The white curve indicates mean zonal flow.

Simplified/intermediate complexity GCMs: Parameter sweeps and scaling

- Mitchell & Vallis [2010] noted that Kelvin waves were only present during spin-up of super-rotation
  - Equlibrated state was dominated by Rossby and MRG waves with only weak Kelvin modes – mechanism…?
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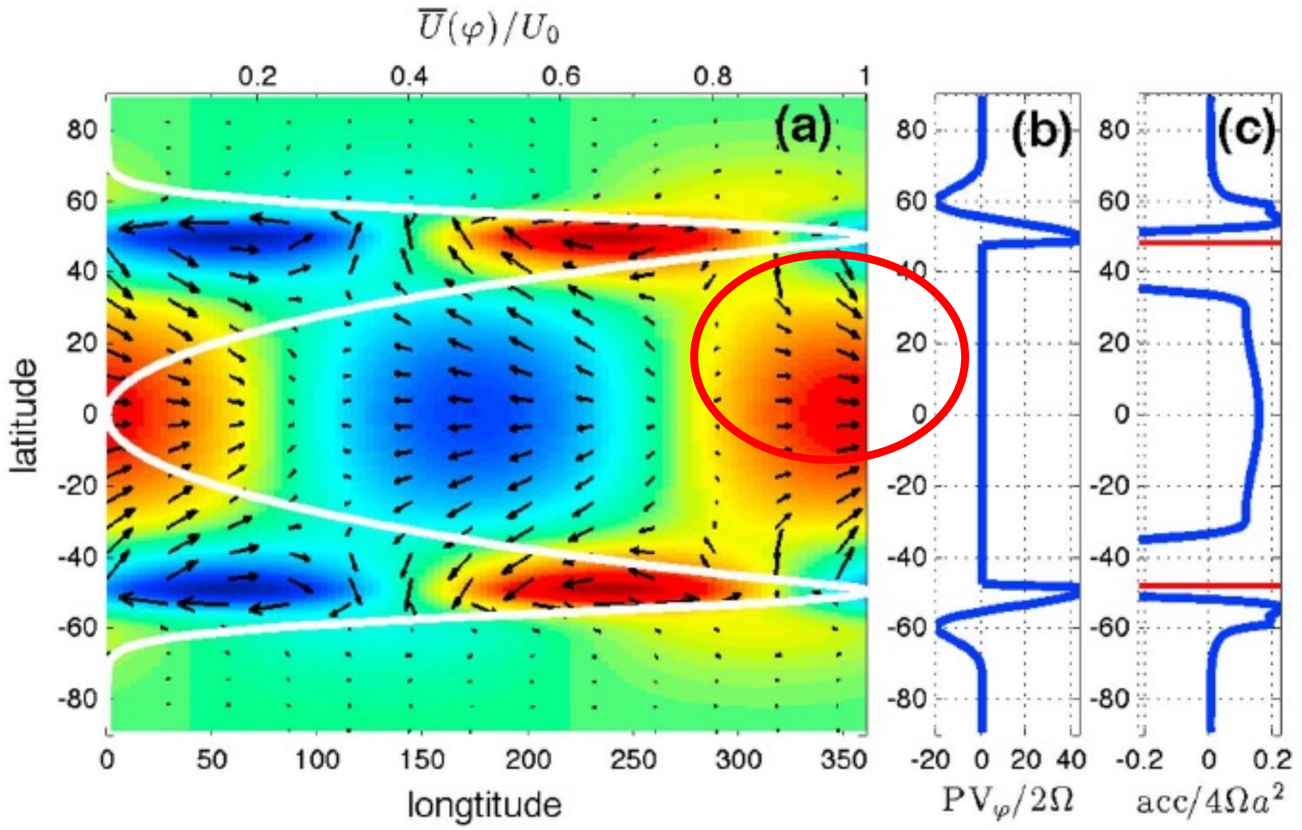
  - Formed by a resonant interaction between a pair of linear equatorial Kelvin and Rossby waves
  - Doppler-shifted by zonal wind



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  - Instability depends on reversal of PV gradients and on Burger no. $\mathcal{B}$ and Froude number

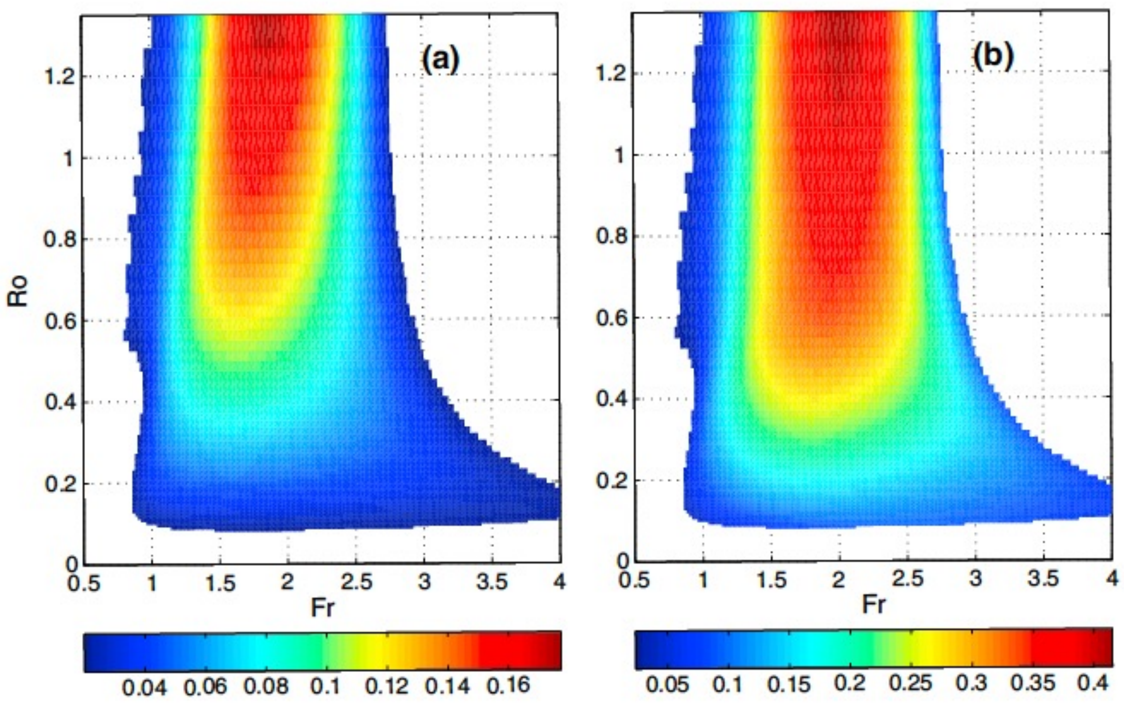

Figure 3. The (a) growth rate σ and (b) zonal momentum acceleration at equator of the Rossby-Kelvin instability on a (*Fr*, *Ro*) space. The growth rate σ is scaled by the rotation rate 2Ω, and the zonal momentum acceleration is scaled by $4a\Omega^2$.

Simplified/intermediate complexity GCMs: Parameter sweeps and scaling – final remarks

- Wave-zonal flow acceleration depends on wave dissipation, forcing or transience
  - Many possible scenarios
  - Acceleration of $\bar{u}$ in direction of c for dissipating waves
  - Critical layer absorption limits $\bar{u}$ to ~c for dissipating waves
  - Acceleration of $\bar{u}$ in opposite direction to c for forced waves (e.g. thermal tides)
  - Strong super-rotation possible with forced waves….
- Many of these mechanisms can be (and have been) demonstrated in lab experiments
- Simple GCMs show some clear trends
  - But numerics need to be interpreted with caution, especially for slow rotators